1985

A Comparison of Leadership Styles Between College and University Administrators of Physical Education in the United States and Korea

Kwang Min Cho

COMMON WORK SPACE OF TWO ROBOT ARMS

BY

BYUNG OH CHOI

A thesis submitted
in partial fulfillment of the requirements for the
degree of Master of Science
Major in Mechanical Engineering
South Dakota State University
1985

COMMON WORK SPACE OF TWO ROBOT ARMS

This thesis is approved as a creditable and independant investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Dr. Bashir A. Sayar — Date

Thesis Adviser

Dr. Lawrence Hooks — Date

Head, Mechanical Engineering Department

ABSTRACT

The choice of kinematic and dynamic parameters for a robot arm depends on the desired working space, desired load lifting capacity, and desired performance of the robot. The inverse process, which is the determination of a single robot's work space from given specifications, is also possible, and is very important both to the designers and to the users. Proceeding further, the analysis of the common work space of two robot arms expands our understanding of spatial limitations of the communicating, cooperative robots of the future.

This thesis presents a new possible application of industrial robots in which two robots work together interactively, or individually, within their common work space. Kinematic equations of anthropomorphic articulated robot arms are derived by using homogeneous 4 x 4 transformation matrices. Based on these equations, the possible working boundaries of two-revolute and three-revolute jointed robot arms are defined.

Based on the geometric influence coefficients, the common working areas and volumes of two-link and three-link robot arms are investigated and calculated. The

calculations and plotting of these areas and volumes are performed on a digital computer.

The results reached show that the common working areas and volumes of two fixed identical robots depend on the distance between their bases. Maximum common working area (face-to-face orientation) occurs at an optimum distance, but the maximum common working volume occurs when the two robots are at the exact same position. Realistically, this means the robots should be placed as near to each other as practical if it is desired to maximize their common working volume.

ACKNOWLEDGEMENTS

My sincere gratitude is extended to all who have helped me finish this study. In particular, I am grateful to Dr. Bashir A. Sayar for his support of my work and his critical reviews and kind guidance.

I remain grateful to Dr. Lawrence Hooks, Head of the Mechanical Engineering Department, for his moral and financial support. I also remain grateful to other professors in Mechanical Engineering for being so receptive when I needed their help. I am also obliged to Miss Kathy McKinley for her continuous guidance of my life in U.S.A..

My special appreciation is due to my parents and relatives for their financial support and encouragement to make my M.S study possible. I would like to express my appreciation to my wife, Yang Ock, for her encouragement, patience, and initiative in taking care of our home while I was a student here.

Appendix page

LIST OF FIGURES

Chapter I

INTRODUCTION

To increase productivity, improve quality, reduce job hardship, and reduce cost, modern manufacturers find themselves increasingly in favor of utilizing robots in the production industries. Unlike automated machinery, robots can be moved from any position and orientation to any other location, and can be reprogrammed conveniently. The robot itself has restrictions on the tasks it can perform. The arrangements of the links and joint axes will affect the ability of the robot arm to manipulate objects in the complex work space environment.

A number of studies have contributed to our understanding of the robot geometry and work space design which have also been applied to industrial robots. Roth {1} was the first to present a study relating the kinematic parameters of a manipulator with its work space. Gupta and Roth {2} introduced some basic concepts regarding the work space of manipulators. They discussed in detail the shapes of the work space and the effect of hand size.

Tsai and Soni {3,4,5} studied the work space of two and three-link robots. They determined the accessible work regions of two-link and three-link robot arms {3}. They also presented a procedure to synthesize two and three-link robot arms. In addition, they presented a method to plot the work spaces of three revolute, 3-R, robot arms which can make a complete rotation {4}. They also studied the effect of link parameters on both the shape and the volume of the work space. Finally, they developed an algorithm to determine the work space of a N revolute, N-R, robot arms on an arbitrary plane {5}. Their algorithm is based on a linear optimization technique and on small incremental displacements applied to coordinate transformation equations relating the kinematic parameters of the N-R robot. This algorithm provides flexibility to let the user treat the robot hand as a point, a line, or a rigid-body.

Kumar and Waldron {6} developed the algorithm for tracing the boundary surfaces of the work space. Yang and Lee {7} derived a set of recursive equations in terms of motion and design parameters representing the work space. These formulas are basic for the determination of the characteristics, as well as the shape, of the work space. Yang and Lee also investigated the existance of holes and voids in the work space. They further introduced a manipulator performance index, and proved that for a given manipulator structure the ratio of the volume of the work space to the cube of its total link length is a constant

{8}. They also presented algorithms for outlining the boundary profile of the work space, and for quantitative evaluation of its volume.

Kohli and Spanos {9,10} developed a new method of work space analysis of manipulators by using polynomial discriminants, and applied their new method to a number of regional structures of robots. Sugimoto and Duffy {11,12} presented their theories for determining the extreme distances of manipulator's hand.

Cwiakala and Lee {13} presented an algorithm using optimization techniques to outline the boundary profile of a manipulator work volume. In particular, they used the 3 x 3 dual-number matrix method as the basis for their analytical formulations. Freudenstein and Primrose {14} analyzed the work space of a three-axis, articulated robot arm of general proportions in terms of the volume swept out by the surface of a skew torus rotating about an offset axis in space.

This thesis research includes an extension of work space studies reported earlier for a single robot arm, plus an in-depth analysis of the common work space of two robot arms. The expanded work on this area may help both robot designers and users for their clear understanding of common work space.

Chapter two gives a general background of kinematic equations of robot arms. Homogeneous 4x4 rotation and translation transformation matrices {15} are used to represent various configurations of robot links. For serial robot arms, the specification of Ai matrices, which describes the relative position and orientation of each link, is introduced. The product of these makes the homogeneous transformation matrix which describes the absolute position and orientation of the end of a robot arm. The method of solving transformation matrix is also introduced.

In chapter three, kinematic equations are derived for two-link and three-link robots using a basic transformation method which is reviewed in chapter two. Using these equations, computer programs are developed to draw the boundaries of the working areas (planar case), to calculate the working areas and volumes, and to find the effect of link parameters.

In chapter four, the interacting work space of two robot arms is analyzed in a basic manner. The programs developed draw the boundaries of the common work space and calculate the volume that is generated by rotating the base of the robot. Synthesis of two robot arms to perform a common task within the common work space is discussed in this chapter. Chapter five presents some possible future applications, and chapter six contains the conclusions of this study.

Chapter II

KINEMATIC EQUATIONS OF A ROBOT ARM

In this chapter a general review of kinematic equations of a robot arm is presented. Any robot arm consisting of a series of links connected together by joints can be considered. From these kinematic constraints, kinematic equations which are purely geometric can be found by using the calculus and trigonometric relationships.

2.1 THE TRANSFORMATION MATRIX

The transformation matrix can be used to express the kinematic equations of a robot arm. By embedding coordinate frames in each link of the robot, transformation matrices (Ai) may be obtained relating the ith link coordinate frame to the (i-1)st link coordinate frame. The transformation from the base frame to the gripper of the robot hand is given by:

$$Ti = A1 \cdot A2 \cdot A3 \cdot \cdots \cdot A(i-1) \cdot Ai \tag{2-1}$$

where i is the number of links.

The general form of the transformation matrix is

$$Ti = \begin{bmatrix} Xx & Yx & Zx & Px \\ Xy & Yy & Zy & Py \\ Xz & Yz & Zz & Pz \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2-2}$$

2.2 SPECIFICATION OF AI MATRICES

The transformation matrices are functions of the joint and link parameters. The Ai matrices are homogeneous transformation matrices describing the relative translation and rotation between links. Therefore, the A1 matrix describes the position and orientation of the first link. The A2 matrix describes the position and orientation of the second link with respect to the first link. Thus, the absolute position and orientation of the second link is obtained by the matrix product. The Ai matrix can be defined as the product of translation and rotation matrices.

$$Ai = (\text{translations})\ (\text{rotations}) \tag{2-3}$$

Based on the geometric and trigonometric relations, translation and rotation matrices can be derived from Figure 1.

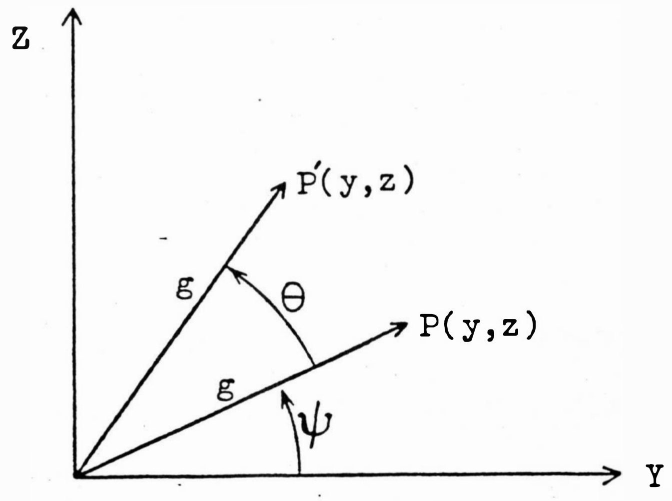


Figure 1. Matrix Rotation

Rotating by θ in Figure 1 shows the transformation of P(Y,Z) to P′(Y,Z). The distance from the origin to P and to P′ are equal and are labeled g in Figure 1.

$$Yp = g\cos(\psi)$$
$$Zp = g\sin(\psi) \tag{2-4}$$

$$Yp' = g\cos(\psi+\theta) = g\cos(\psi)\cos(\theta) - g\sin(\psi)\sin(\theta)$$
$$Zp' = g\sin(\psi+\theta) = g\cos(\psi)\sin(\theta) + g\sin(\psi)\cos(\theta). \tag{2-5}$$

Substitute Equation (2-4) into Equation (2-5) to obtain Equation (2-6)

$$Yp' = Yp\cos(\theta) - Zp\sin(\theta)$$
$$Zp' = Yp\sin(\theta) + Zp\cos(\theta). \tag{2-6}$$

In 2 X 2 matrix form, one can get Equation (2-7) as:

$$\begin{bmatrix} Y_p' \\ Z_p' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} Y_p \\ Z_p \end{bmatrix}. \qquad (2\text{-}7)$$

The above matrix, Equation (2-7), presents the transformation of the Y-Z plane in two dimensions. If one considers the three-dimensional case, one can treat Equation (2-7) as a matrix rotation about the X axis through an angle of α degrees (Figure 3). From these relationships, one can construct homogeneous 4x4 transformation matrices.

From Figure 2, the basic translation matrix can be defined in terms of

$$\text{Trans}\ (Px,Py,Pz) = \begin{bmatrix} 1 & 0 & 0 & Px \\ 0 & 1 & 0 & Py \\ 0 & 0 & 1 & Pz \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2\text{-}8)$$

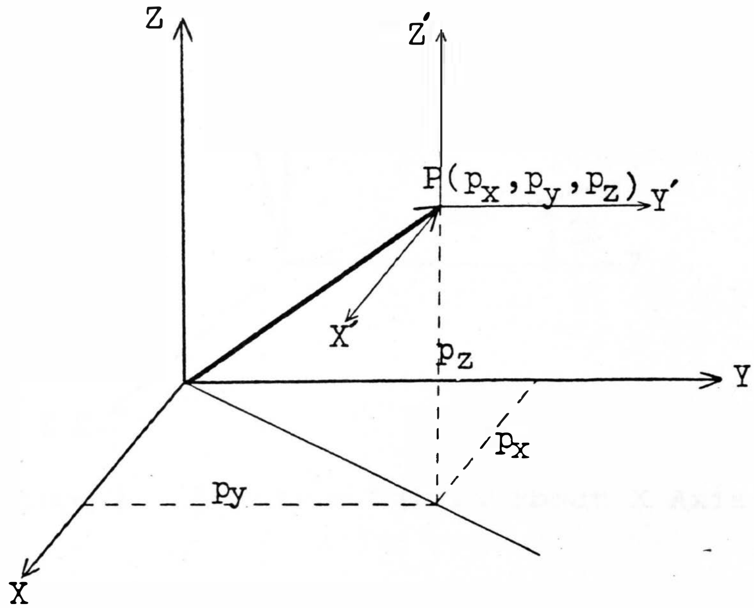


Figure 2. Translation Matrix

In the general translation matrix, the fourth column elements represent the translated position with respect to the original coordinates. This translation matrix simply moves the origin to point P(Px,Py,Pz) without any change of the coordinates orientation.

From Figures 3, 4, and 5, the basic rotation matrices can be defined in terms of

$$\mathrm{Rot}\ X(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2\text{-}9)$$

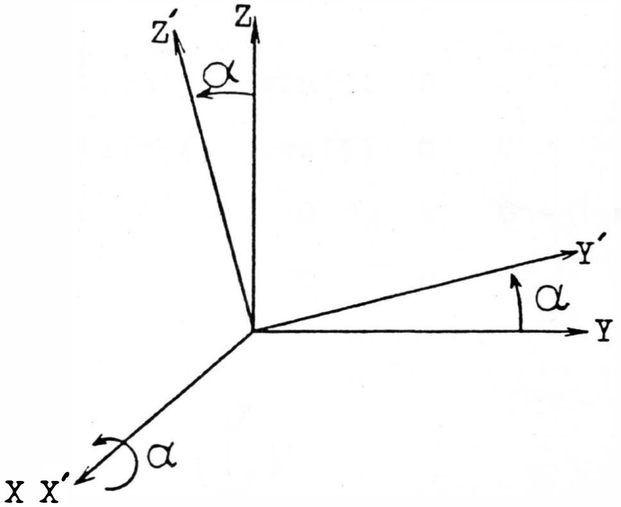


Figure 3. Rotation Matrix about X Axis

$$\text{Rot Y}(\beta) = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2\text{-}10)$$

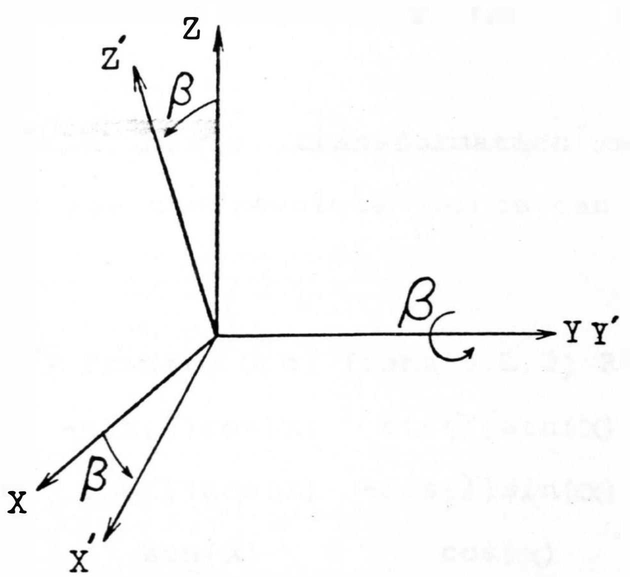


Figure 4. Rotation Matrix about Y Axis

$$\text{Rot } Z(\gamma) = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2\text{-}11)$$

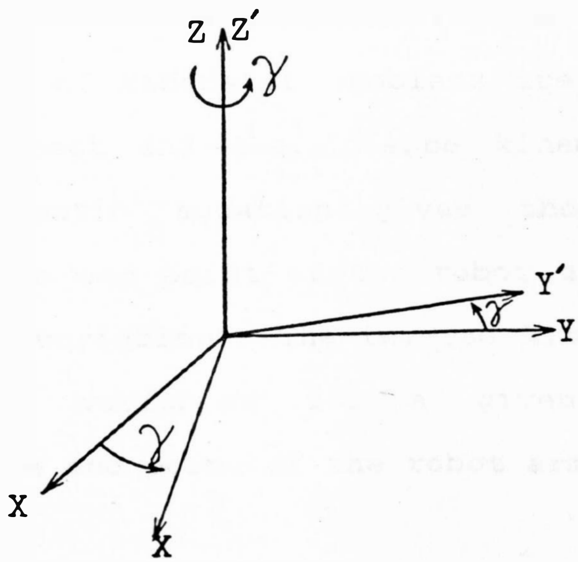


Figure 5. Rotation Matrix about Z Axis

Using those basic transformation matrices, the general Ai matrix for the revolute joints can be defined in terms of

$$Ai = \text{Rot } Z(\gamma)\ \text{Trans}(0,0,d)\ \text{Trans}(0,L,0)\ \text{Rot } X(\alpha)$$

$$= \begin{bmatrix} \cos(\gamma) & -\sin(\gamma)\cos(\alpha) & \sin(\gamma)\sin(\alpha) & -L\sin(\gamma) \\ \sin(\gamma) & \cos(\gamma)\cos(\alpha) & -\cos(\gamma)\sin(\alpha) & L\cos(\gamma) \\ 0 & \sin(\alpha) & \cos(\alpha) & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2-12)

in which d = distance between the links (joint offset)

α = angle of rotation about the X axis

L = the common normal distance (link length)

γ = angle of rotation about the Z axis

2.3 SOLUTION OF THE TRANSFORMATION MATRIX EQUATION

Two kinds of kinematic problems are encountered in robotics; the direct and the inverse kinematic problems. The direct kinematic equation gives the position and orientation of the end point of the robot arm given values of all the joint variables. The inverse kinematic equation gives the joint variables for a given position and orientation of the end point of the robot arm.

2.3.1 The Direct Kinematic Equation

For a given joint angle of a robot arm, the forward kinematic equation can simply be obtained by evaluating the transformation matrices Ai to obtain Ti.

$$Ti = A1 \cdot A2 \cdot A3 \cdot \cdots \cdot Ai = \begin{bmatrix} X & Y & Z & P \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2\text{-}13)$$

In Ti, the vector P(Px,Py,Pz) is the position of the robot arm end with respect to the base coordinates. As for

X, Y, and Z, they are the projections of the base unit vectors of the end position coordinate frame onto the base coordinate frame. They can be used to obtain the orientation and the position of a robot arm end.

2.3.2 The Inverse Kinematic Equation

Obtaining a solution for the joint coordinates is of the utmost importance in robot control. To use robots on the production lines, one normally knows where one wants to move the robot arm in terms of T6. One therefore needs to obtain the joint coordinates of the robot arm in order to make the move.

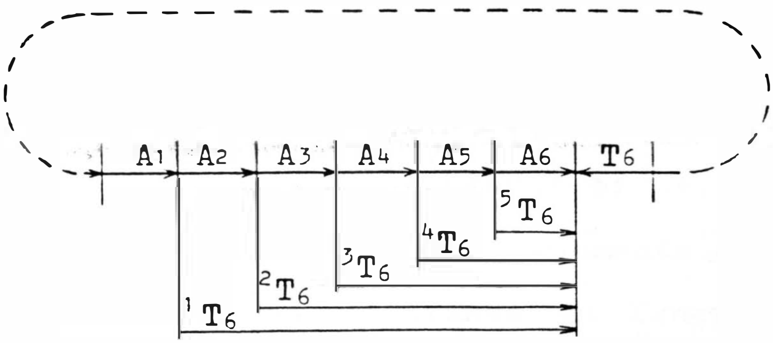


Figure 6. Manipulator Transform Graph

For industrial robots having six degrees of freedom, T6 is known and is equal to the product of the six Ai matrices (see Figure 6)

$$T6 = A1 \cdot A2 \cdot A3 \cdot A4 \cdot A5 \cdot A6 \qquad (2\text{-}14)$$

Six matrix equations are then obtained by successively premultiplying Equation (2-14) by the inverses of the Ai matrices.

$$A1^{-1} \cdot {}^{0}T6 = {}^{1}T6$$

$$A2^{-1} \cdot A1^{-1} \cdot {}^{0}T6 = {}^{2}T6$$

$$A3^{-1} \cdot A2^{-1} \cdot A1^{-1} \cdot {}^{0}T6 = {}^{3}T6$$

$$A4^{-1} \cdot A3^{-1} \cdot A2^{-1} \cdot A1^{-1} \cdot {}^{0}T6 = {}^{4}T6$$

$$A5^{-1} \cdot A4^{-1} \cdot A3^{-1} \cdot A2^{-1} \cdot A1^{-1} \cdot {}^{0}T6 = {}^{5}T6 \qquad (2\text{-}15)$$

in which the subscripts refer to the Nth joint, and the superscripts refer to the reference joint.

The matrix elements of the left hand sides of these equations are functions of the elements of T6, and of the first N-1 joint variables. The matrix elements of the right hand sides are either zeros, constants, or functions of the Nth to 6th joint variables. As matrix equality applies element by element, one can obtain 12 equations from each matrix equation. Solving those 12 equations, one can then obtain each joint angle of the robot arm for a given position and orientation. However, the inverse kinematics analysis as described above is very difficult and cumbersome to handle.

Chapter III

TWO-LINK AND THREE-LINK ROBOT ARMS

Most industrial robots have six degrees of freedom to match the requirements of rigid body motion. From the kinematics point of view, the structure of such robots can be treated as a combination of two structures, which are "regional structure" and "orientational structure". The regional structure, which consists of the shoulder and arms, contributes to the gross location of the robot hand. The orientational structure, which consists of the wrist and hand, contributes to the orientation of the robot hand.

To analyze the characteristics of the work space of a robot, one can simply study the work space of its regional structure. One needs the equations which describe the end point of the robot arm to define the work space of the robot. Any robot can be considered to consist of a series of links connected by joints. Using the 4 x 4 homogeneous transformation matrices, one can describe the relative position and orientation between the joints.

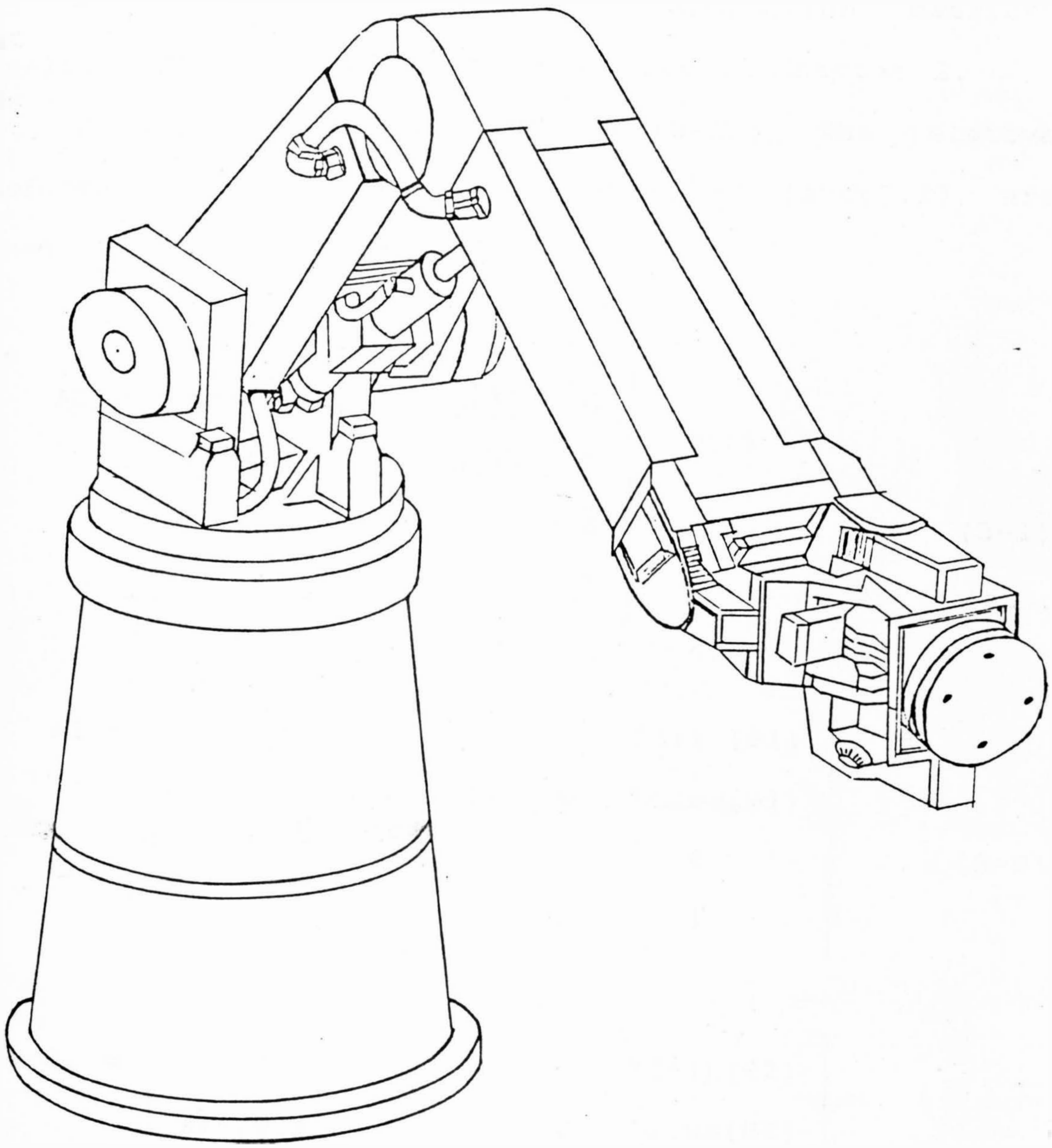


Figure 7. General Industrial Robot

3.1 TWO-LINK ROBOT ARMS

3.1.1 The Transformation Matrix of Two-Link Robot Arms

One can derive the kinematic equations of two-link robot arms using the basic transformation matrices (translation, rotation) which are defined in Chapter 2.

From Figure 8 and Equation (2-12), the relative transformation matrices for the links, Ai (i=0,1,2), are defined as:

$$A0 = \begin{bmatrix} \cos(\gamma) & 0 & \sin(\gamma) & 0 \\ \sin(\gamma) & 0 & -\cos(\gamma) & 0 \\ 0 & 1 & 0 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3-1}$$

$$A1 = \begin{bmatrix} \cos(\theta 1) & \sin(\theta 1) & 0 & L1\sin(\theta 1) \\ -\sin(\theta 1) & \cos(\theta 1) & 0 & L1\cos(\theta 1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3-2}$$

$$A2 = \begin{bmatrix} \cos(\theta 2) & \sin(\theta 2) & 0 & L2\sin(\theta 2) \\ -\sin(\theta 2) & \cos(\theta 2) & 0 & L2\cos(\theta 2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3-3}$$

Therefore, the T2 matrix can be obtained by taking the product of the Ai (i=0, 1, 2) matrices as in the following:

$$T2 = A0 \cdot A1 \cdot A2$$

$$= \begin{bmatrix} \cos(\gamma)\cos(\theta1+\theta2) & \cos(\gamma)\sin(\theta1+\theta2) & \sin(\gamma) & \cos(\gamma)(L1\sin(\theta1) + L2\sin(\theta1+\theta2)) \\ \sin(\gamma)\cos(\theta1+\theta2) & \sin(\gamma)\sin(\theta1+\theta2) & -\cos(\gamma) & \sin(\gamma)(L1\sin(\theta1) + L2\sin(\theta1+\theta2)) \\ -\sin(\theta1+\theta2) & \cos(\theta1+\theta2) & 1 & L1\cos(\theta1) + L2\cos(\theta1+\theta2) + d \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3-4)$$

The T2 matrix as shown above defines the relative link locations for kinematic analysis and orientation of the end position of the two-link robot arm. In the T2 transformation matrix, the 4th column elements indicate the end point coordinates of the robot arm. The nine elements of the first, second, and third columns and rows, represent the orientation of the end point of two-link robot arm.

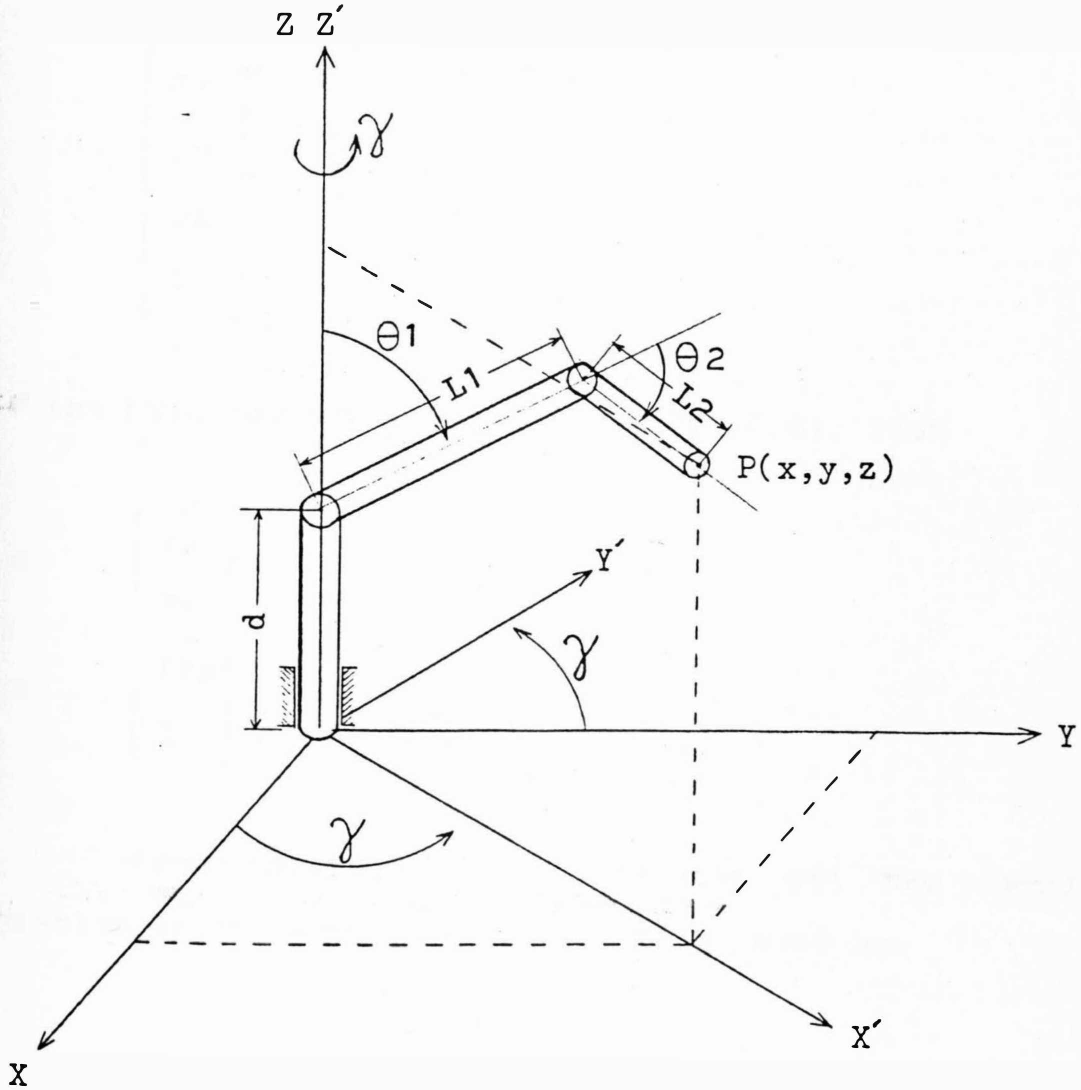


Figure 8. Two-Link Robot Arm

3.1.2 The Work Space of Two-Link Robot Arms

From the T2 transformation, the end point coordinate is P(Px,Py,Pz) with respect to base coordinate O(Xo,Yo,Zo)

$$\begin{bmatrix} Px \\ Py \\ Pz \\ 1 \end{bmatrix} = T2 \cdot \begin{bmatrix} Xo \\ Yo \\ Zo \\ 1 \end{bmatrix} \qquad (3\text{-}5)$$

If the base coordinates are zero, O(0,0,0), then

$$\begin{bmatrix} Px \\ Py \\ Pz \\ 1 \end{bmatrix} = T2 \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (3\text{-}6)$$

From Equation (3-6), one can get the coordinate equation of the end point of the robot arms as:

$$\begin{aligned} Px &= (L1 \sin(\theta 1) + L2 \sin(\theta 1+\theta 2)) \cos(\gamma) \\ Py &= (L1 \sin(\theta 1) + L2 \sin(\theta 1+\theta 2)) \sin(\gamma) \\ Pz &= L1 \cos(\theta 1) + L2 \cos(\theta 1+\theta 2) + d \end{aligned} \qquad (3\text{-}7)$$

From Equation (3-7), the Z coordinate is not affected by the base rotation γ, therefore, if γ is set equal to 90 degrees, then Equation (3-8) will result

$$
\begin{aligned}
P_x &= 0 \\
P_y &= L_1 \sin(\theta_1) + L_2 \sin(\theta_1+\theta_2) \\
P_z &= L_1 \cos(\theta_1) + L_2 \cos(\theta_1+\theta_2) + d
\end{aligned} \tag{3-8}
$$

These equations describe the robot arm manipulation in the Y-Z plane and the axes of joints therefore are parallel to the X axis.

Using Equation (3-8), one can draw the boundary line of the work space in the Y-Z plane. Four extreme combinations of the angles θ_1 and θ_2 can be obtained. These combinations will generate the boundary of the work space of a robot in the following manner, and as illustrated in Figure 9

1. $\widehat{AB}$ can be drawn from $\theta_1 = (\theta_1)_{min}$ and $(\theta_2)_{min} < \theta_2 < (\theta_2)_{max}$
2. $\widehat{BC}$ can be drawn from $\theta_2 = (\theta_2)_{max}$ and $(\theta_1)_{min} < \theta_1 < (\theta_1)_{max}$
3. $\widehat{AD}$ can be drawn from $\theta_2 = (\theta_2)_{min}$ and $(\theta_1)_{min} < \theta_1 < (\theta_1)_{max}$
4. $\widehat{DC}$ can be drawn from $\theta_1 = (\theta_1)_{max}$ and $(\theta_2)_{min} < \theta_2 < (\theta_2)_{max}$

In order to get the work volume, one should rotate the base of the robot so as to produce the volume of revolution of the work area around the Z axis. It should be noted that the work space of the revolute-jointed robot is symmetrical about the Z axis. Figure 10 shows the shape of the work space of a two-link robot arm in the Y-Z and X-Y planes.

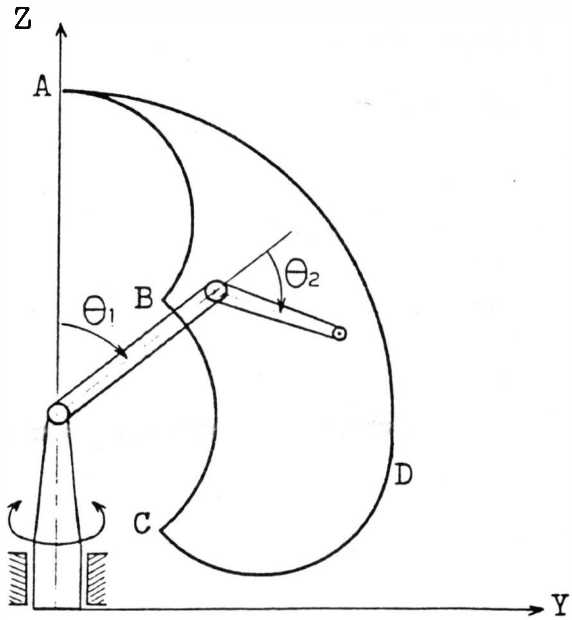


Figure 9. Boundary Contour of the Work Space of Two-Link Robot Arms

3.1.3 Plotting the Work Space of a Two-Link Robot Arm

After obtaining the end coordinate equation of a robot arm, one can draw the boundary line of the work space by evaluating the four extreme cases and the base rotation due to small angle increments. In order to draw the work space in the X-Y plane, one should select an arbitrary Z coordinate. A typical example of such a work space projection is the area ABCDA in plane PXY of Figure 11. Figure 11 also shows another typical plane, PZY, in which the vertical projection of the work space is shown as EFGHE. The pictorial sketch of the work space volume of the two-link robot arms is shown in Figure 12.

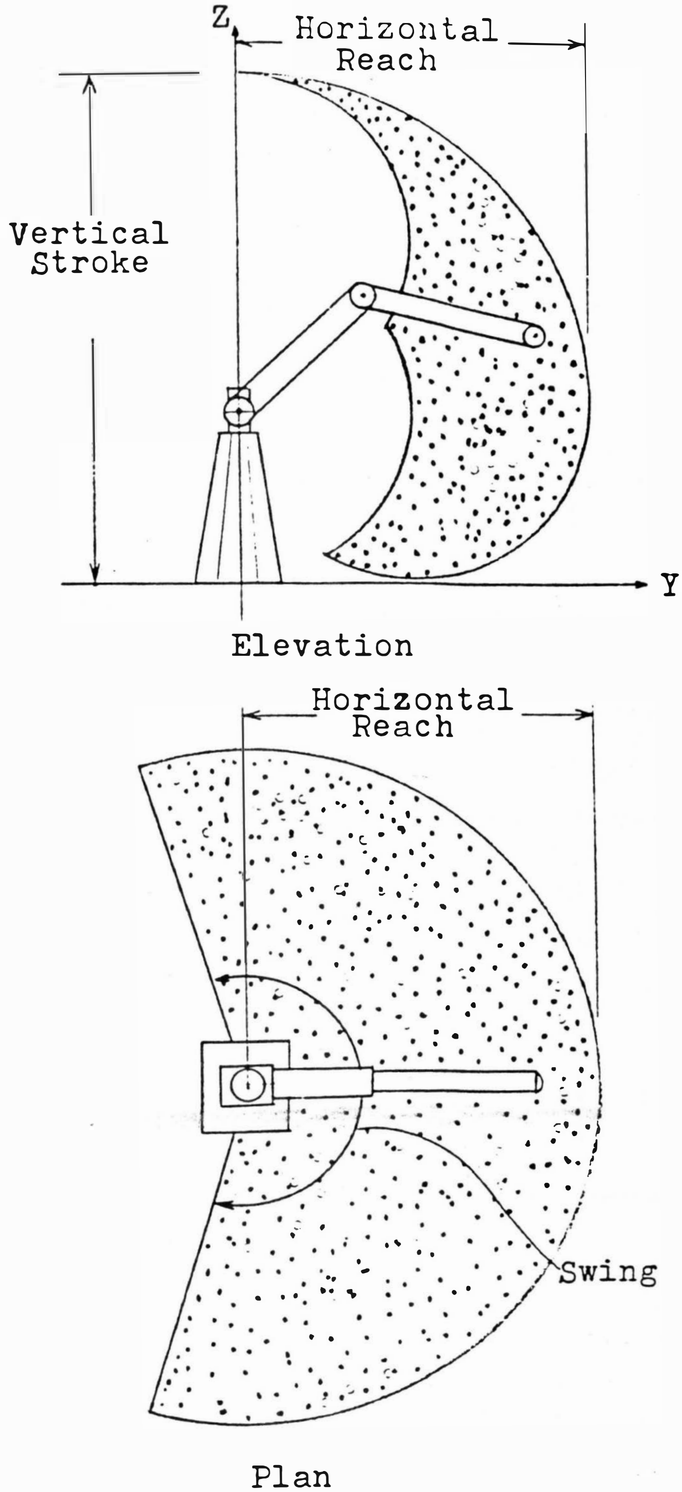


Figure 10. Contour of the Work Space of Two-Link Robot Arms

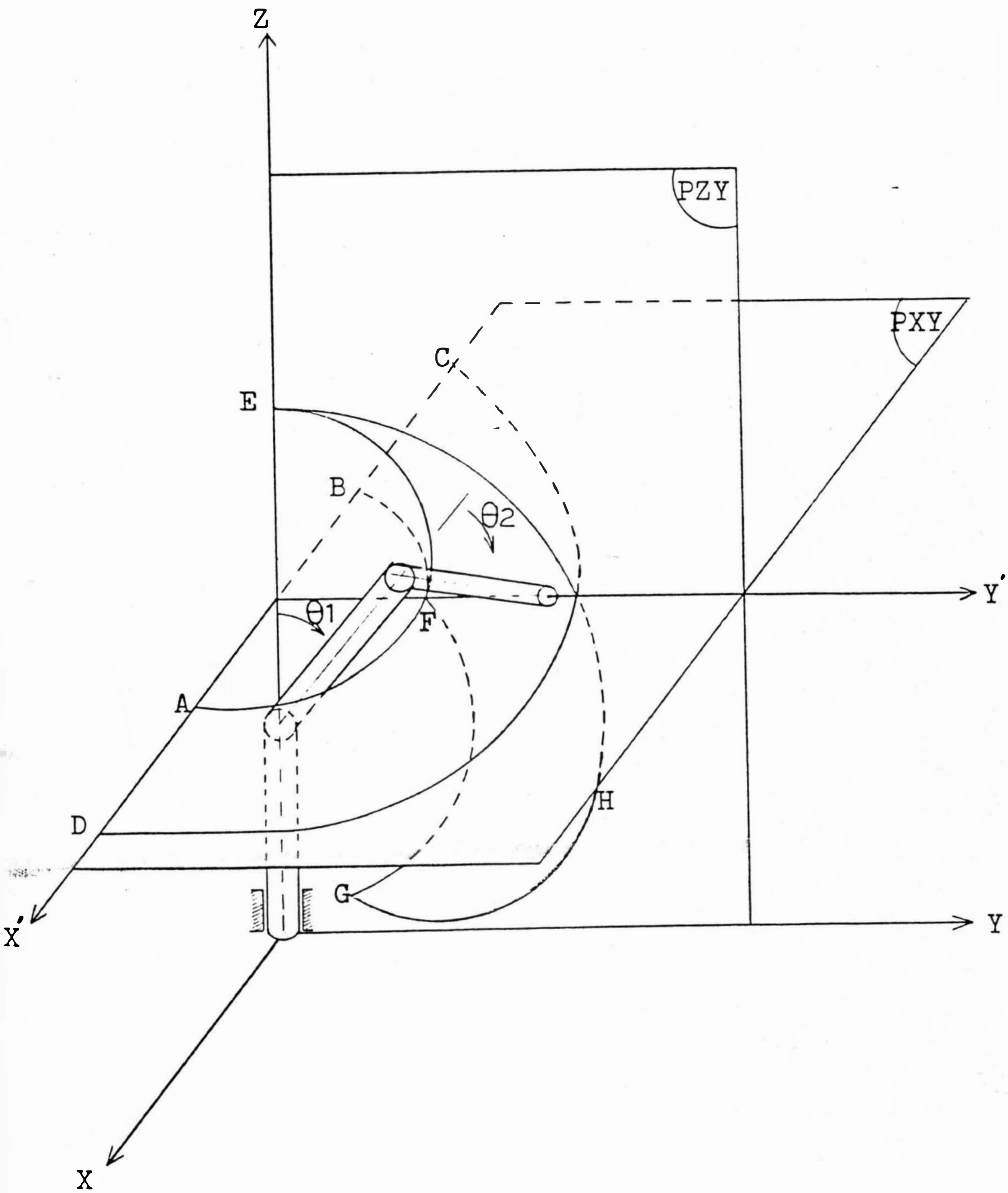


Figure 11. Typical Planes for the Plotting the Work Space

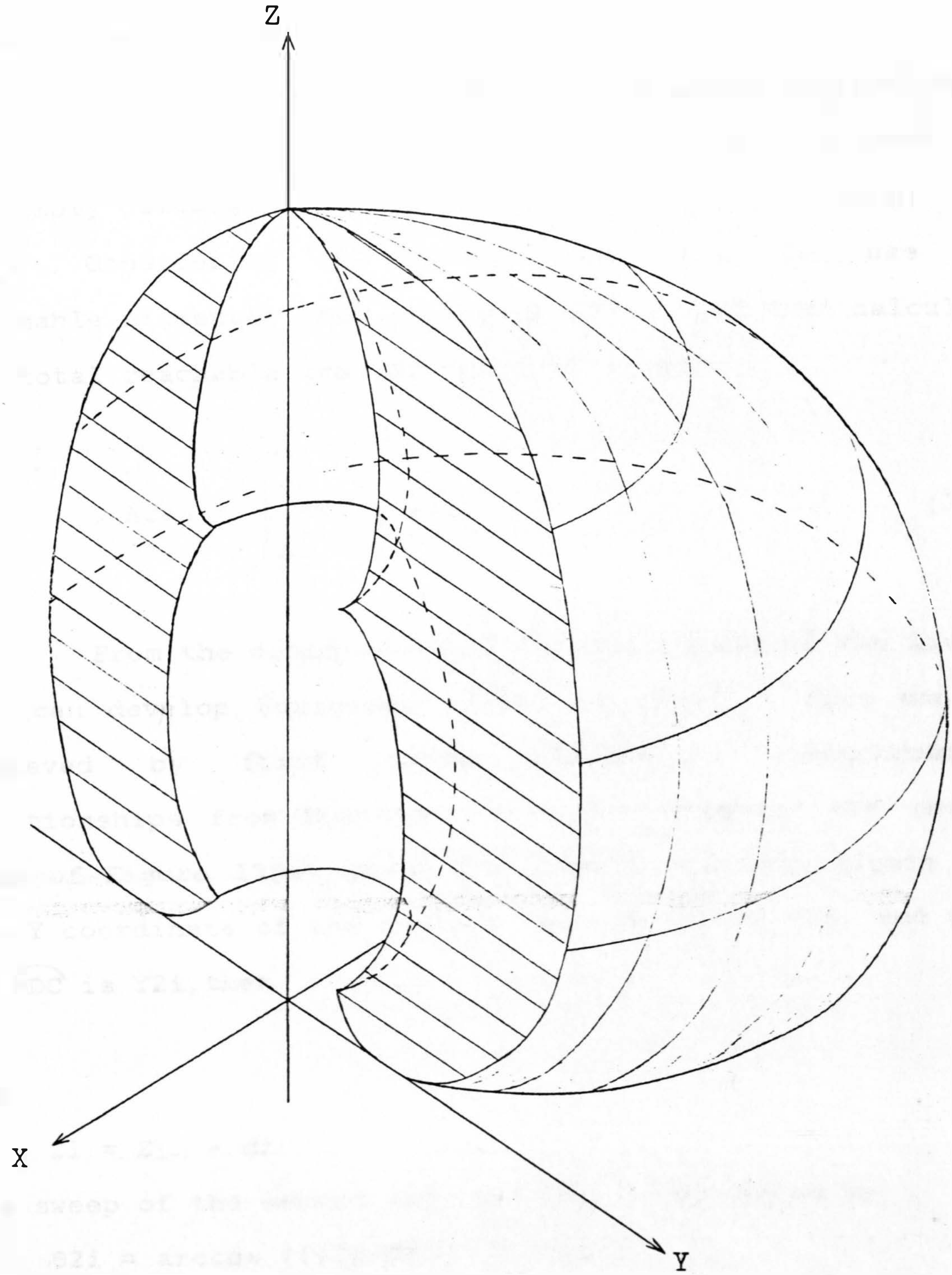


Figure 12. Sketch of the Work Space of Two-Link Robot Arms

3.1.4 Analysis of the Work Space Volume

Knowing the boundary of the work space projection on a plane, say the Y-Z plane, the volume of the work space can be simply calculated by the Euler's integration method.

Considering the planar case, one can use the reachable sigmental area of Figure 13(a), and then calculate the total reachable area by Equation (3-9).

$$\text{Area} = \int_{yz} F(0,y,z)\,dz \tag{3-9}$$

From the coordinates of the end points of the links, one can develop Equations (3-10) to (3-13). This can be achieved by first using geometrical trigonometric relationships from Equation (3-8) to integrate the shaded area of Figure 13(a) about the Z axis. If from Figure 12, the Y coordinate of the contour line of $\overset{\frown}{ABC}$ is Y1i, and that of $\overset{\frown}{ADC}$ is Y2i, then

for

$$Zi = Z_{i-1} - dz$$

the sweep of the second arm, arc $\overset{\frown}{AB}$, is generated by

$$\theta 2i = \arccos\left(((Zi-d) - L1)/L2\right)$$

with

$$\theta 1i = (\theta 1)\min, \quad i = 0, 1, 2, 3, \ldots.$$

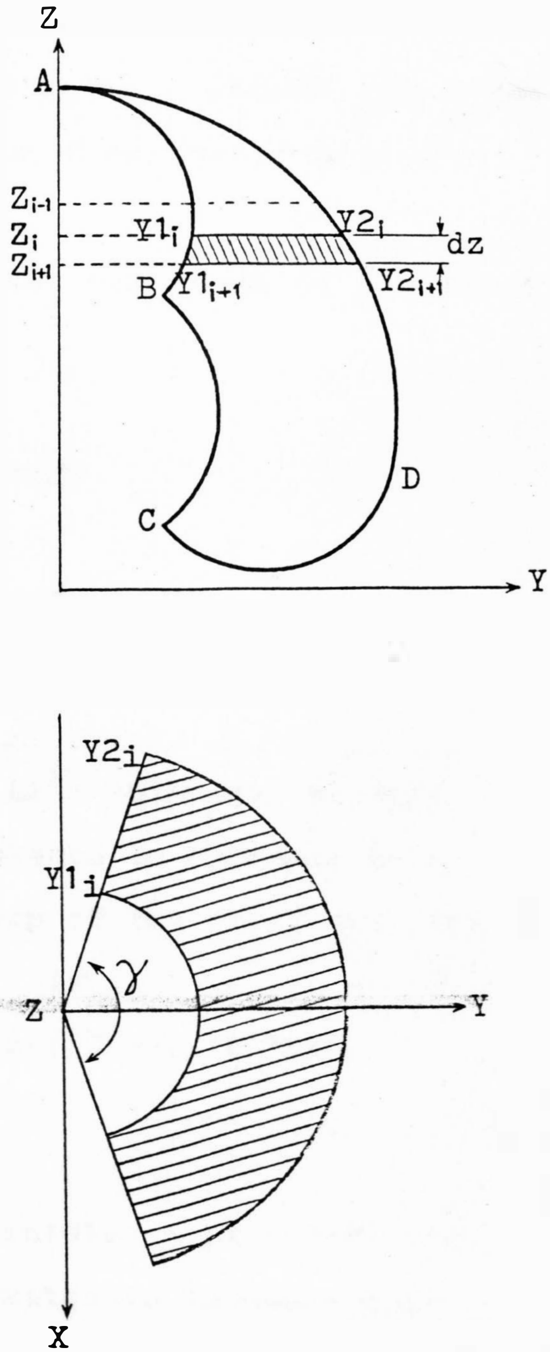


Figure 13. Evaluation of the Work Space Volume

Therefore,

$$Y1i = L1\ \sin(\theta 1i) + L2\ \sin(\theta 1i+\theta 2i) \qquad (3\text{-}10)$$

The configurational sketch to explain the above relationships is shown in Appendix A-1.

if $\theta 2i = (\theta 2)max$, the sweep of the first arm, arc $\widehat{BC}$, will result in

$$Y1i = \rho 1 \sin(\phi) \qquad (3\text{-}11)$$

in which

$$\phi = \arccos((Zi-d)/\rho 1)$$

and by the cosine law,

$$\rho 1 = \sqrt{L1^2 + L2^2 + 2L1L2\cos((\theta 2)max)}$$

(see the sketch in Appendix A-2)

The sweep of the first arm, arc $\widehat{AD}$, when $O2i=O$, $i=O, 1, 2, \ldots$ is

$$\theta 1i = \arccos((Zi-d)/(L1+L2))$$

leading to:

$$Y2i = L1\ \sin(\theta 1i) + L2\ \sin(\theta 1i+\theta 2i) \qquad (3\text{-}12)$$

(see the sketch in Appendix A-1)

However, for the generation of arc $\widehat{DC}$

$$\theta 1i = (\theta 1)max, \text{ and}$$

$$Y2i = L1\sin((\theta 1)max) + \sqrt{L2^2 - M^2} \qquad (3\text{-}13)$$

with

$$M = ((Zi-d) - L1\cos((\theta 1)max))$$

(see the sketch in Appendix A-3)

From Equation (3-9) and Equations (3-10) to (3-13)

$$\text{Area} = \int_{Zmin}^{Zmax} (Y2i - Y1i) \, dz \qquad (3\text{-}14)$$

with dz representing the average height of the incremental rectangle. Applying Euler's integration method, one can write:

$$(\text{Area})i = A_{i-1} + (Y2i - Y1i) \, dz$$

Now, to calculate the volume of the work space

$$\text{Volume} = \int_{xy} \int_{yz} f(x,y,z) \, dz \, dx \qquad (3\text{-}15)$$

If one considers the work space projection in the X-Y plane, as shown in Figure 12(b), the area of the shaded plane can be written as

$$A(X,Y,Zj) = (Y2i^2 - Y1i^2)\cdot\pi\cdot\gamma/2\cdot\pi$$

$$= (Y2i^2 - Y1i^2)\cdot\gamma/2 \qquad (3\text{-}16)$$

for which γ is the rotation of the base of the robot about the Z axis at a constant elevation, Zj. Once again, the volume can be calculated by integrating Equation (3-16);

$$\text{Volume} = \int_{Zmin}^{Zmax} A(X,Y,Z)\, dz \qquad (3\text{-}17)$$

In fact, one can use an alternative integration method to calculate the work area and volume for the revolute jointed industrial robot shown in Figure 7. This alternative method is explained and illustrated in Appendix B.

The computer programs to perform the integrations in Equations (3-14), (3-16), (3-17) are written in BASIC, and are developed using Euler's integration method. Although the Euler's integration method is not as accurate as the other methods, it is quite stable. Fortunately, the shape of the work space of most industrial robots is coplanar, and therefore the error due to the uncounted end shape will be much smaller than expected. In fact, the shape of work space is more important than its magnitude in application.

As a matter of convenience, the unit of each dimension in all calculations will be U (Unit), so one can use any customary units such as feet, meter etc.

3.1.5 Effect of Link Parameters

Because the work space is affected by so many parameters and variables, it is very difficult to analyze the effects of all parameters on the work space at the same time. The effect of link parameters on the work space of two-link robots may be studied in two ways; one is the change of link length ratios, and the other is the change of the limitations of the joint displacement angles.

For two-link robot arms, the shape, area, and volume of the accessible region depend on the variables $L1$, $L2$, $(\theta 1)min$, $(\theta 1)max$, $(\theta 2)min$ and $(\theta 2)max$. To investigate the effect of link parameters, one can first set the θ values, and then change the ratio of $L1$ and $L2$ to analyze the effect of the link length ratio.

Figure 14 shows the effect of the ratio of $L2$ and $L1$ on the area for three combinations of different joint displacements.

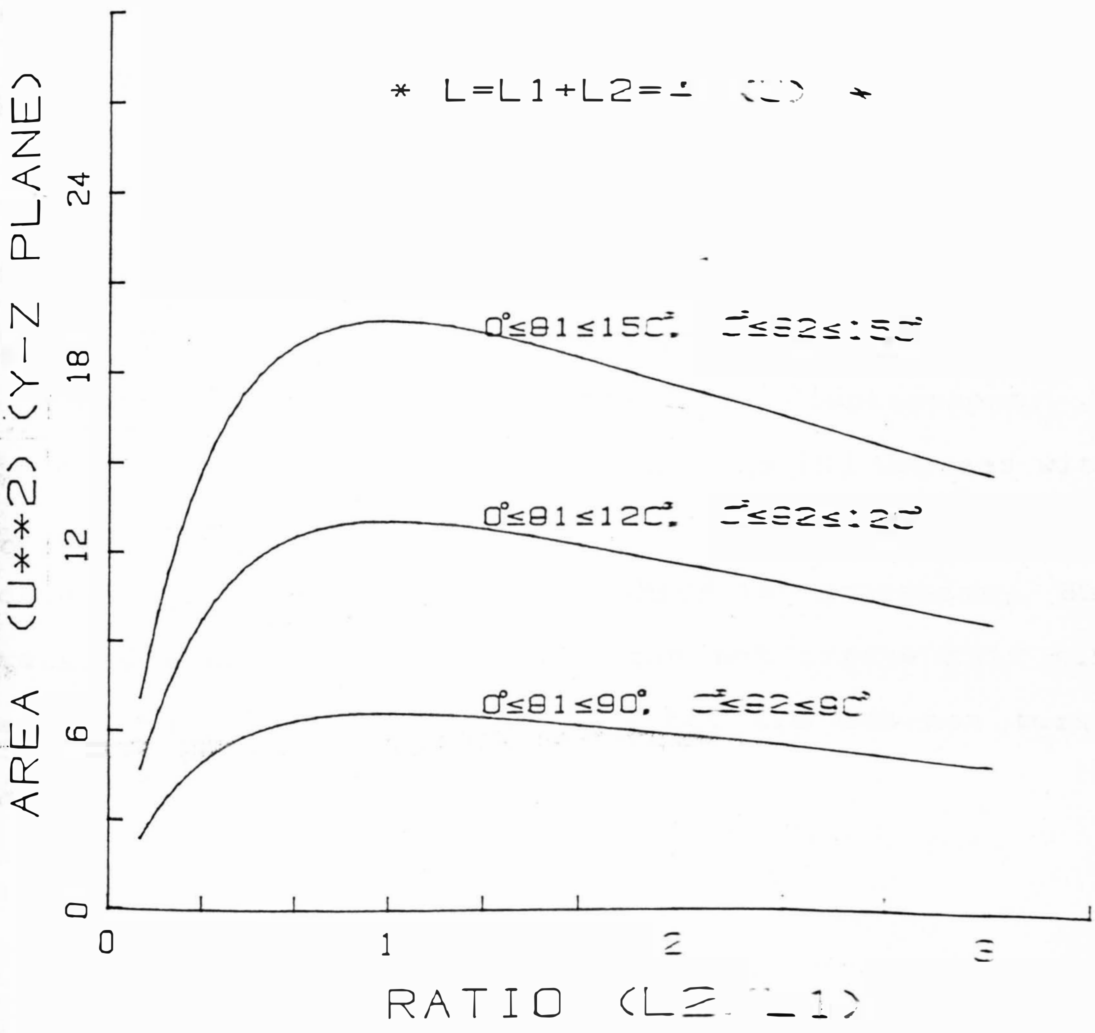


Figure 14. Effect of the Ratio of Link Length

From this figure one can see that the area on the Y-Z plane will be a maximum when the ratio of L1 and L2 is one. Now we know that equal length of two links gives us maximum area and volume.

To analyze the effect of joint displacement, one can set the link length ratio equal to one, prescribe one joint displacement, and calculate the working area on Y-Z plane due to changing the other joint displacement.

Figure 15 shows the change of area in the Y-Z plane for a variation in the second link joint displacement. From this figure, one can observe the increase in the area on the Y-Z plane due to increasing the joint displacement. It is interesting to note that after reaching 180 degrees with the second joint displacement, the extra working space for an offset jointed-link is created which is unnecessary. But the rotated elbow joint robot arm can not create this kind of extra work space, because the robot arm can not turn more than 180 degrees.

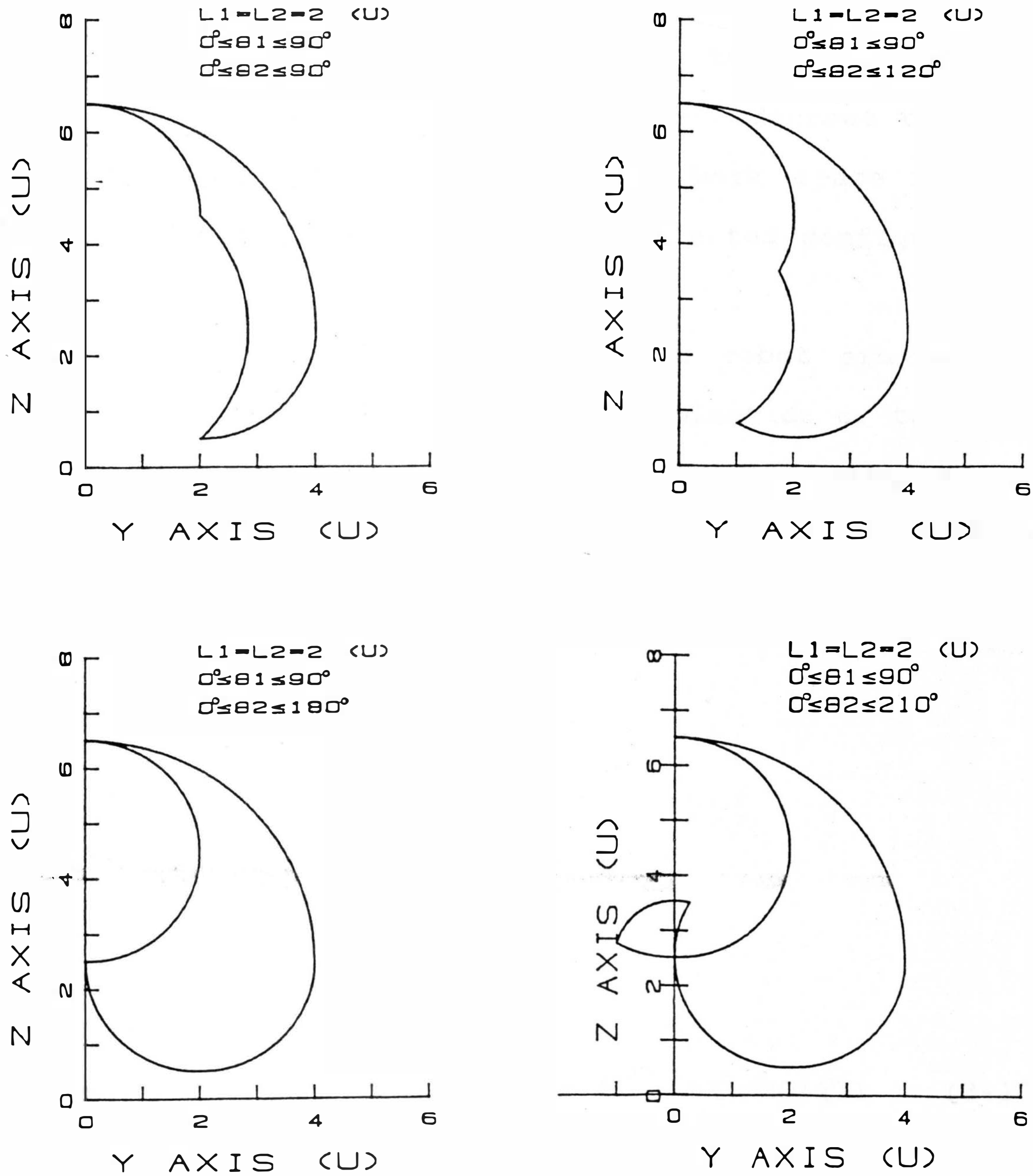


Figure 15. Effect of the Joint Angles (due to θ2)

Now, set the second link joint displacement, and change the first link joint displacement. Then the area on the Y-Z plane will be larger due to increasing the corresponding joint displacement. At 360 degrees of first link displacement, the shape of the work space will be circular. Figure 16 shows the four selected configurations for the above case.

The rotated elbow joint of the robot arm can not turn more than 180 degrees, and the combination of two joint displacements has certain restrictions. For example, the robot hand may hit its own body, or the floor. Therefore, the limits of each joint and combination to avoid having the robot hit itself are as follows,

1) Individual limitation

$(\theta 1)max < 180$ degree

$(\theta 2)max < 180$ degree

2) Combination limitation

$$(\theta 1)max < 180 - \arccos((L1^2 + \rho 1^2 - L2^2)/2L1\rho 1) \quad (3\text{-}18)$$

where

$$\rho 1 = \sqrt{L1^2 + L2^2 + 2L1\ L2\cos((\theta 2)max)}$$

(see the configurational sketch in Appendix A-2)

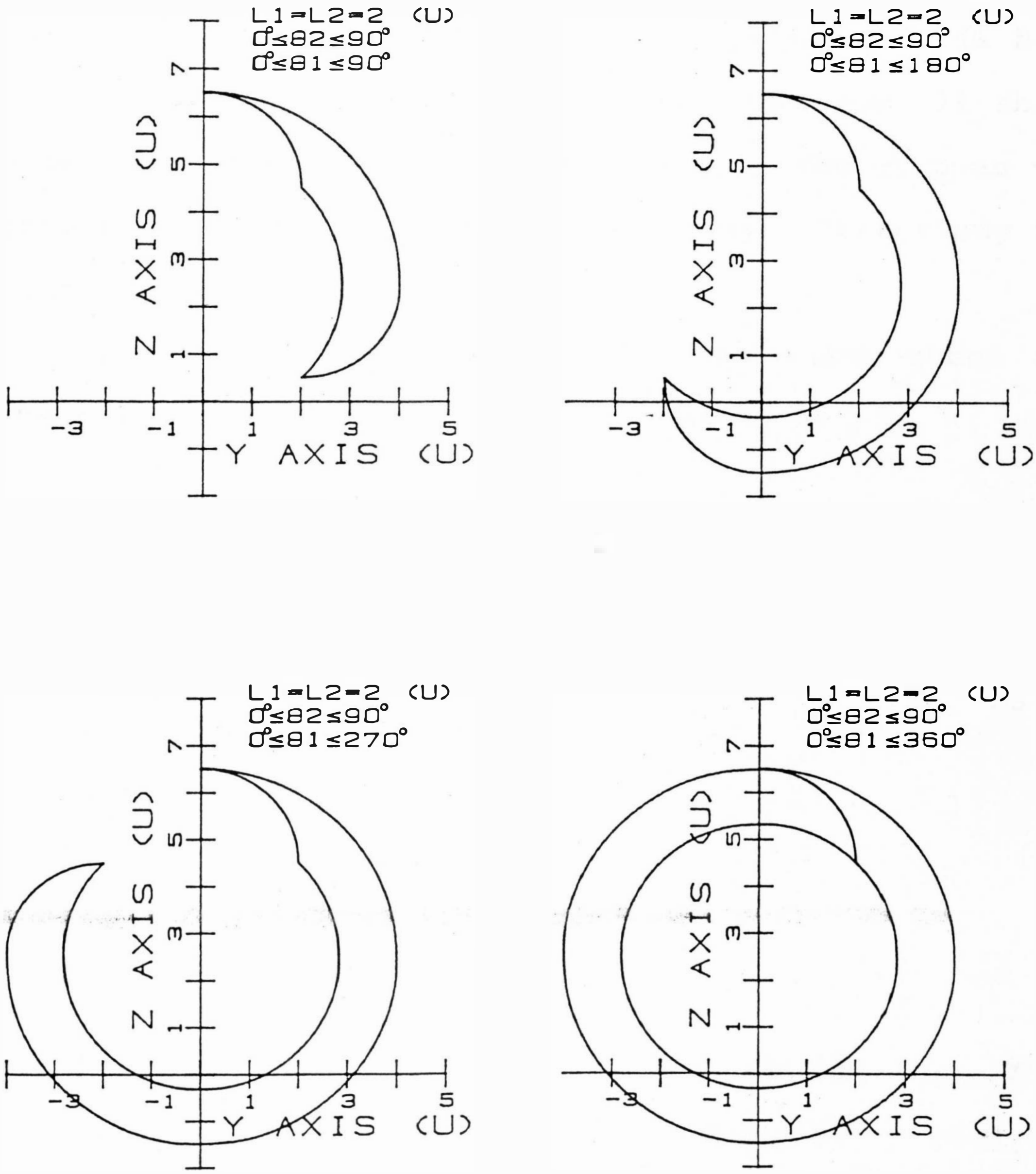


Figure 16. Effect of the Joint Angles (due to θ1)

3.2 THREE-LINK ROBOT ARMS

The regional structure of industrial robots consists of the shoulder and arms, but sometimes the gripper, which is defined to be from the wrist to the center of the hand, can be considered the third link of the robot arm. It should be noted that the pitch and yaw motions of the gripper will contribute to the shape of the work volume. This study does not include such variations.

The transformation matrix of three-link robot arms can be obtained by multiplying T2 by the A3 matrix.

$$A3 = \begin{bmatrix} \cos(\theta 3) & \sin(\theta 3) & 0 & L3\sin(\theta 3) \\ -\sin(\theta 3) & \cos(\theta 3) & 0 & L3\cos(\theta 3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3\text{-}19)$$

$$T3 = T2 \cdot A3 = A0 \cdot A1 \cdot A2 \cdot A3$$

$$T3 = \left[\begin{array}{ccc} \cos(\gamma)\cos(\theta 1+\theta 2+\theta 3) & \cos(\gamma)\sin(\theta 1+\theta 2+\theta 3) & \sin(\gamma) \\ \sin(\gamma)\cos(\theta 1+\theta 2+\theta 3) & \sin(\gamma)\sin(\theta 1+\theta 2+\theta 3) & -\cos(\gamma) \\ -\sin(\theta 1+\theta 2+\theta 3) & \cos(\theta 1+\theta 2+\theta 3) & 1 \\ 0 & 0 & 0 \end{array} \right.$$

$$\begin{bmatrix} \cos(\gamma)(L1\sin(\theta1) + L2\sin(\theta1+\theta2) + L3\sin(\theta1+\theta2+\theta3)) \\ \sin(\gamma)(L1\sin(\theta1) + L2\sin(\theta1+\theta2) + L3\sin(\theta1+\theta2+\theta3)) \\ L1\cos(\theta1) + L2\cos(\theta1+\theta2) + L3\cos(\theta1+\theta2+\theta3) + d \\ 1 \end{bmatrix} \tag{3-20}$$

3.2.1 The Work Space of Three-Link Robot Arms

From the transformation matrix (T3), one can get the coordinates of the end points of three-link robot arms. The end point coordinates, P(Px,Py,Pz), are of the form

$$\begin{aligned} Px &= (L1\sin(\theta1) + L2\sin(\theta1+\theta2) + L3\sin(\theta1+\theta2+\theta3))\cos(\gamma) \\ Py &= (L1\sin(\theta1) + L2\sin(\theta1+\theta2) + L3\sin(\theta1+\theta2+\theta3))\sin(\gamma) \\ Pz &= L1\cos(\theta1) + L2\cos(\theta1+\theta2) + L3\cos(\theta1+\theta2+\theta3) + d \end{aligned} \tag{3-21}$$

Using Equation (3-21), one can find the work space of the three-link robot arm.

To find the boundary line of the work space of a three-link robot, as shown in Figure 17, one may have six extreme conditions which can be obtained by a similar approach to that used in Section 3.1.2.

These six extreme conditions are the following:

1. set $\theta 1 = (\theta 1)_{min}$, $\theta 2 = (\theta 2)_{min}$ and vary $\theta 3$ from $(\theta 3)_{min}$ to $(\theta 3)_{max}$
2. set $\theta 1 = (\theta 1)_{min}$, $\theta 3 = (\theta 3)_{max}$ and vary $\theta 2$ from $(\theta 2)_{min}$ to $(\theta 2)_{max}$
3. set $\theta 2 = (\theta 2)_{max}$, $\theta 3 = (\theta 3)_{max}$ and vary $\theta 1$ from $(\theta 1)_{min}$ to $(\theta 1)_{max}$
4. set $\theta 2 = (\theta 2)_{min}$, $\theta 3 = (\theta 3)_{min}$ and vary $\theta 1$ from $(\theta 1)_{min}$ to $(\theta 1)_{max}$
5. set $\theta 3 = (\theta 3)_{min}$, $\theta 1 = (\theta 1)_{max}$ and vary $\theta 2$ from $(\theta 2)_{min}$ to $(\theta 2)_{max}$
6. set $\theta 1 = (\theta 1)_{max}$, $\theta 2 = (\theta 2)_{max}$ and vary $\theta 3$ from $(\theta 3)_{min}$ to $(\theta 3)_{max}$

If one performs these six operations by small increments of θ, then the shape of the work space will be obtained as shown as in Figure 18.

In three-link robot case, just as in two-link case, the basic approach involves the planar analysis of work space. This approach starts with the proper cross section of the work space. Numerical methods are then developed to determine its boundary, and subsequently the volume of the work space. The projection of this work space in the Y-Z plane is symmetrical about the X-Z plane.

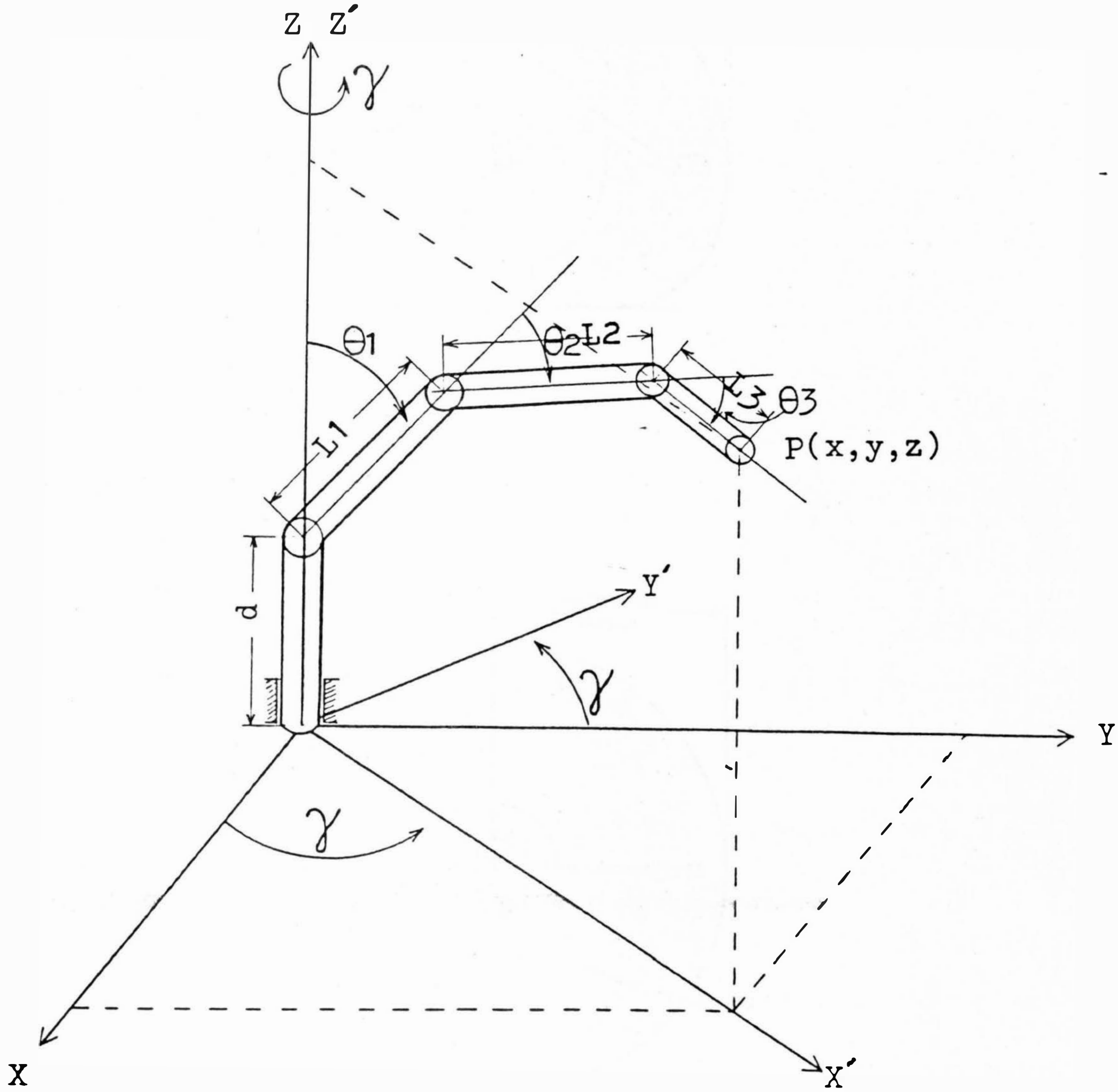


Figure 17. Three-Link Robot Arms

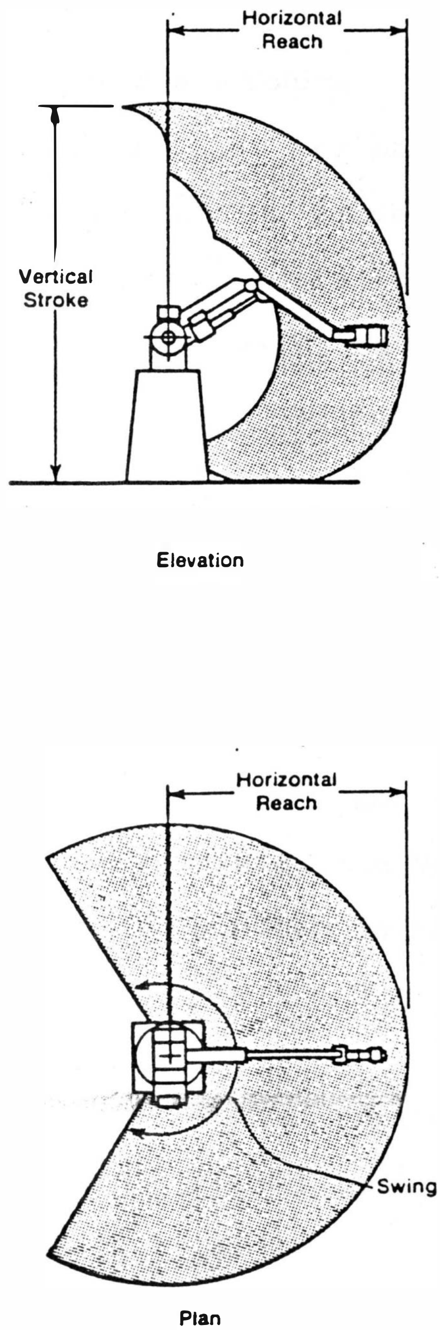


Figure 18. General Shape of the Work Space of a Three-Link Robot

3.2.2 Analysis of the Work Space Volume

The above boundary lines are obtained by evaluating extreme joint variables, such as joint displacements. But to calculate the volume using simple numerical integration, one might have to calculate the Y and Z coordinates for a given boundary point. In other words, Y values in the Y-Z plane will be calculated for a given Z value. Using these Y values, the area in the Y-Z plane will be calculated (refer to Figure 12 and Appendix A).

The algorithm which was developed in article 3.1.4 can also be applied to the three-link case. Based on this algorithm, the BASIC program finds and draws the boundary lines of the work space, calculates the volume, and computes the area of the work space projected on the Y-Z plane.

3.2.3 Effect of Link Parameters

The ratio of link lengths, and the combination of rotating joint angles, are treated here in the same manner as in the case of two-link arms.

The ratio of the link lengths can be evaluated by using the fixed total length of the links. Figure 19 shows the effect of the third link length on the maximum area in the Y-Z plane, which occurs at certain ratios of the third link to the total link length. This link length ratio versus maximum area varies due to the joint displacement variations as shown in Table 3-1. From Figure 19 and Table 3-1, one can see that the link length ratio, giving maximum area on the Y-Z plane, becomes smaller as the joint displacement limits increase.

Figure 20 shows the effect of combinations of L1 and L2 when L3 is fixed. The summary of these combinations is shown in Table 3-2. The ratio of L2 and L1 giving the maximum area on the Y-Z plane becomes larger as the displacement of the joints increases.

Figure 21 shows the effect of the combination of L2 and L3 when L1 is fixed. From Figure 21 and Table 3-3, it is observed that the ratio of L3 and L2 which gives the maximum area in the Y-Z plane becomes smaller as the joint displacements become larger.

Table 3-1 . Effect of Link Length (L3/L)

L=4, L1=L2=(L-L3)/2 (U)

Displacement (degree)	Ratio	Max. Area (U**2)
θ1 = 30 θ2 = 30 θ3 = 30	L3/L = 0.45 (L3=1.8) (L1=L2=1.1)	0.8386
θ1 = 60 θ2 = 60 θ3 = 60	L3/L = 0.444 (L3=1.775) (L1=L2=1.1125)	5.9397
θ1 = 120 θ2 = 120 θ3 = 120	L3/L = 0.40 (L3=1.6) (L1=L2=1.2)	22.8108

Table 3-2 : Effect of Link Length (L2/L1)

L=4, L1+L2=3.5, L3=0.5 (U)

Displacement (degree)	Ratio	Max. Area (U**2)
$\theta 1 = 30$ $\theta 2 = 30$ $\theta 3 = 30$	L2/L1 = 0.59 (L1=2.2) (L2=1.3)	0.5409
$\theta 1 = 60$ $\theta 2 = 60$ $\theta 3 = 60$	L2/L1 = 0.75 (L1=2) (L2=1.5)	3.9793
$\theta 1 = 120$ $\theta 2 = 120$ $\theta 3 = 120$	L2/L1 = 1.333 (L1=1.5) (L2=2)	18.295

Table 3-3 : Effect of Link Length (L3/L2)

L=4, L1=1, L2+L3=3 (U)

Displacement (degree)	Ratio	Max. Area (U**2)
$\theta 1 = 30$ $\theta 2 = 30$ $\theta 3 = 30$	L3/L2 = 2.75 (L2=0.8) (L3=2.2)	0.84
$\theta 1 = 60$ $\theta 2 = 60$ $\theta 3 = 60$	L3/L2 = 2 (L2=1) (L3=2)	5.88
$\theta 1 = 90$ $\theta 2 = 90$ $\theta 3 = 90$	L3/L2 = 1.5 (L2=1.2) (L3=1.8)	14.52

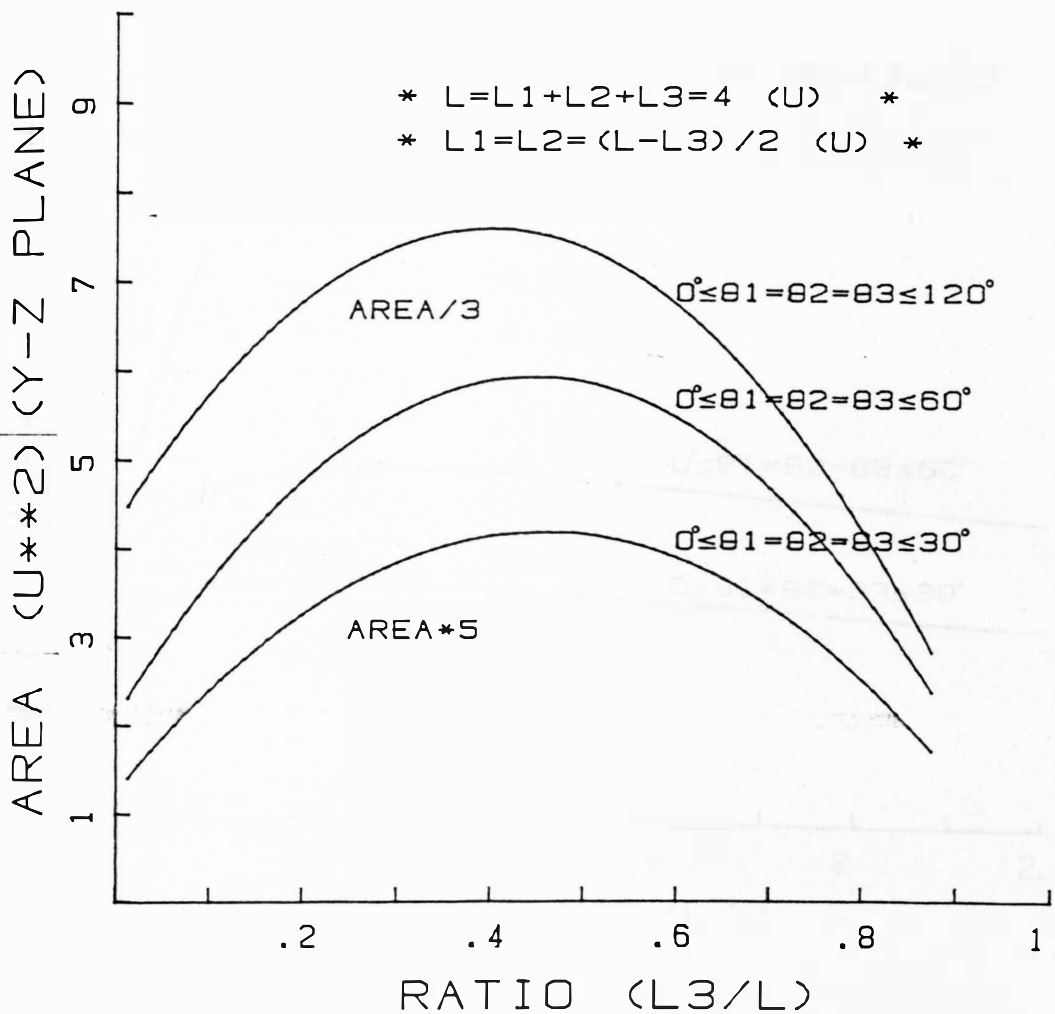


Figure 19. Effect of Link Lengths (L3/L)

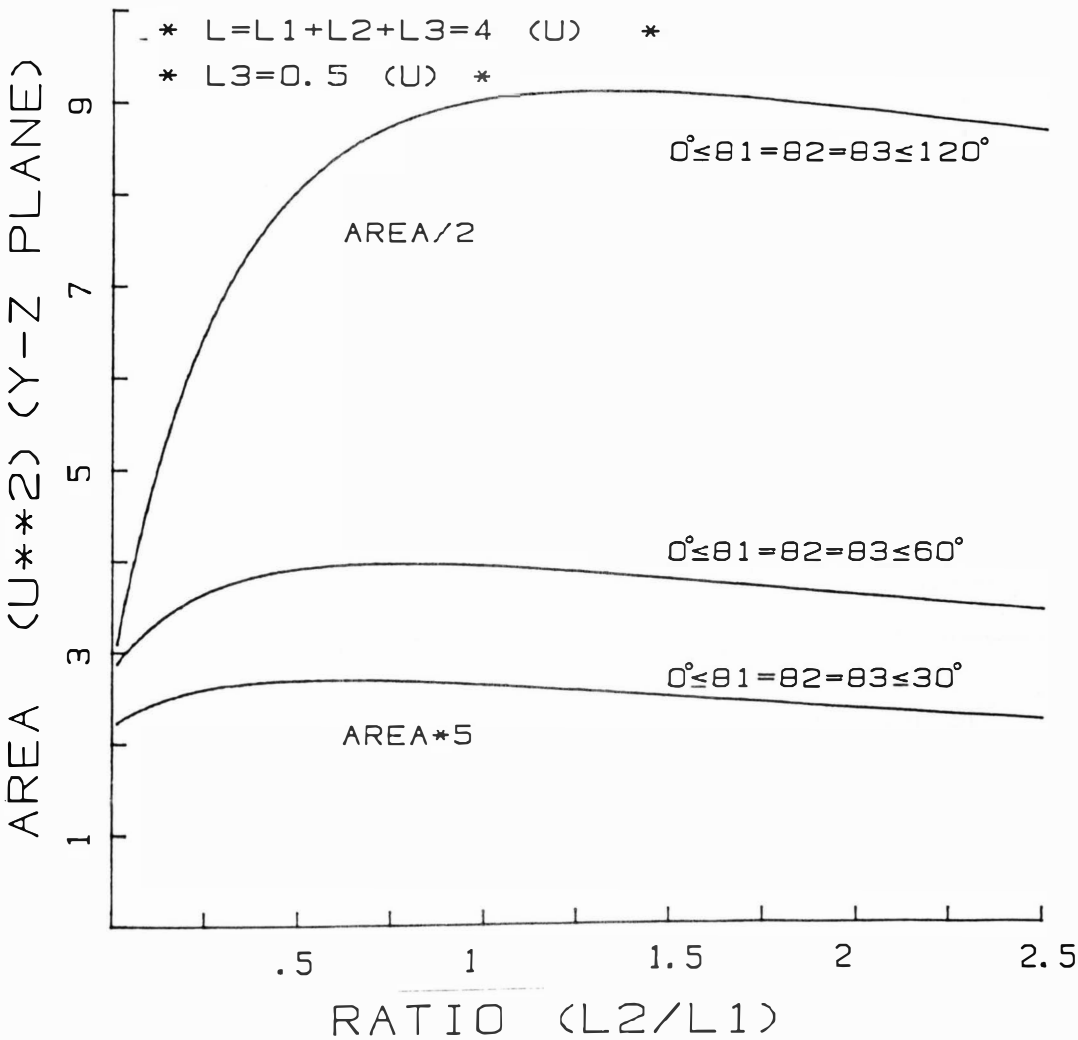


Figure 20. Effect of Link Lengths (L2/L1)

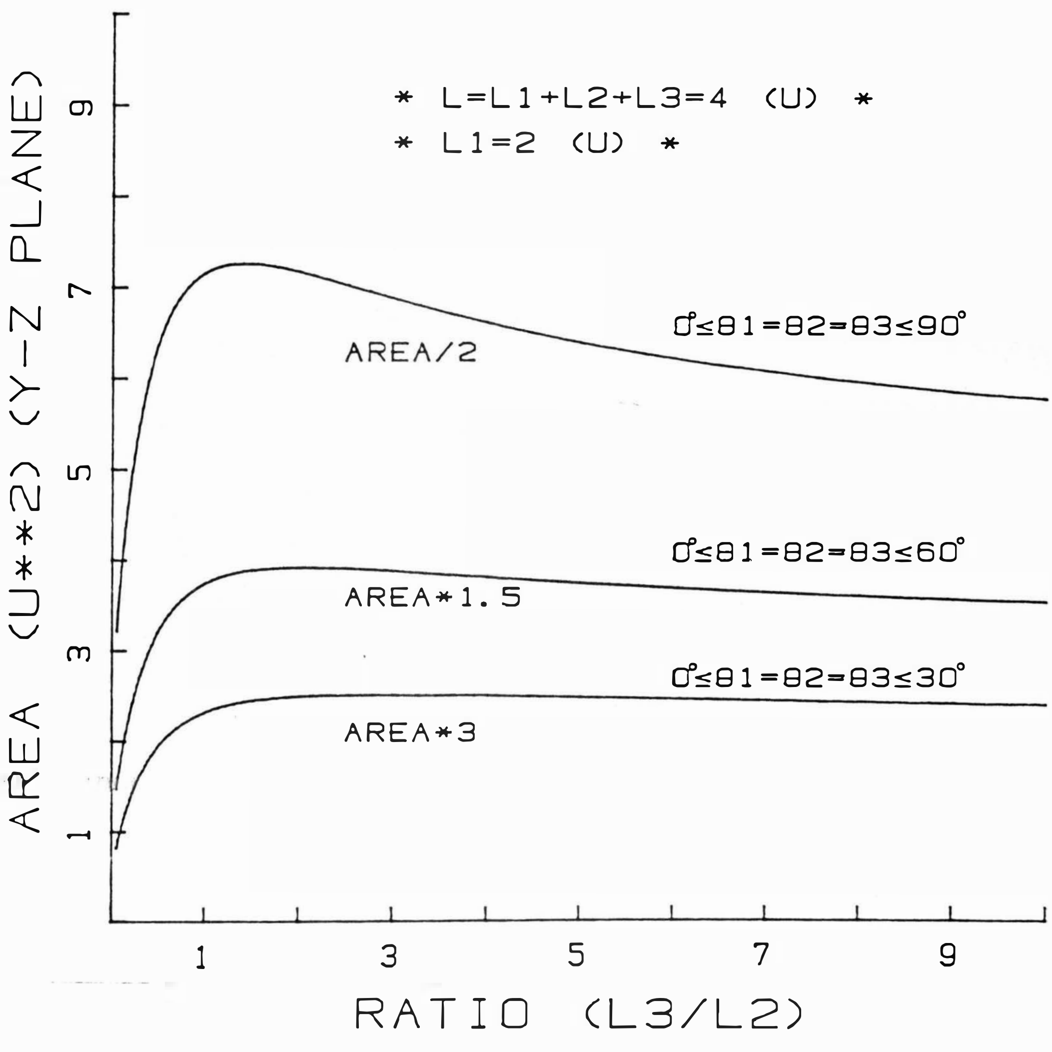


Figure 21. Effect of Link Lengths (L3/L2)

For a given link length, one can evaluate the effect of joint angles by analyzing the work space in terms of joint angles. This is a very complicated process. Furthermore, calculating the work space for given joint displacements is more desirable than finding the joint angles which give the maximum volume in practice.

Figures 22 to 24 show the effect of joint displacements on the projected area of the work space. The second and third links will create the extra work space when the displacement exceeds 180 degrees. The change of joint displacement of the first link does not create extra work space. As the first link displacement is increased to 360 degrees, the shape of the work space becomes circular.

However, the restrictions on joint displacement of the third link and the second link will be the same as in the two-link robot arm case. The joint displacement limit of the first link has the form

$$(\theta 1)max < 180 - \phi$$

with

$$\phi = \arccos((L1^2 + \rho 3^2 - \rho 2^2)/2L1\rho 3)$$

$$\rho 3 = \sqrt{L1^2 + \rho 2^2 + L1\rho 2\cos((\theta 2)max+w)}$$

$$w = \arccos((L2^2 + \rho 2^2 - L3^2)/2L2\rho 2)$$

$$\rho 2 = \sqrt{L2^2 + L3^2 + 2L2L3\cos((\theta 3)max)}$$

(see the configurational sketch in Appendix A-4)

Therefore, the limits of the joint displacement of each link will be related to each other. The joint displacement, and the lengths of the links, if within the specified limits, will prevent the arm from striking the robot itself.

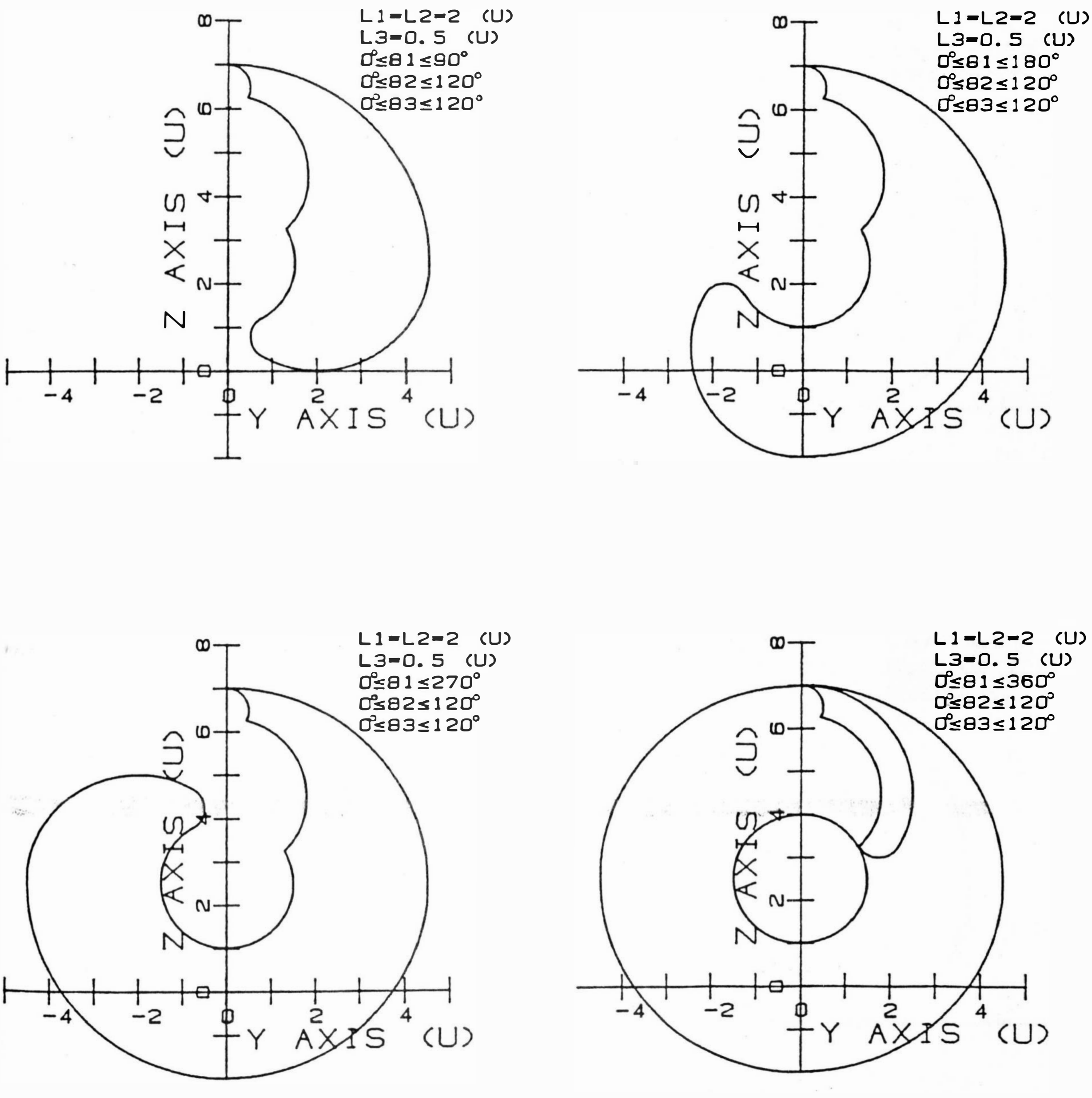


Figure 22. Effect of Joint Displacement (due to θ1)

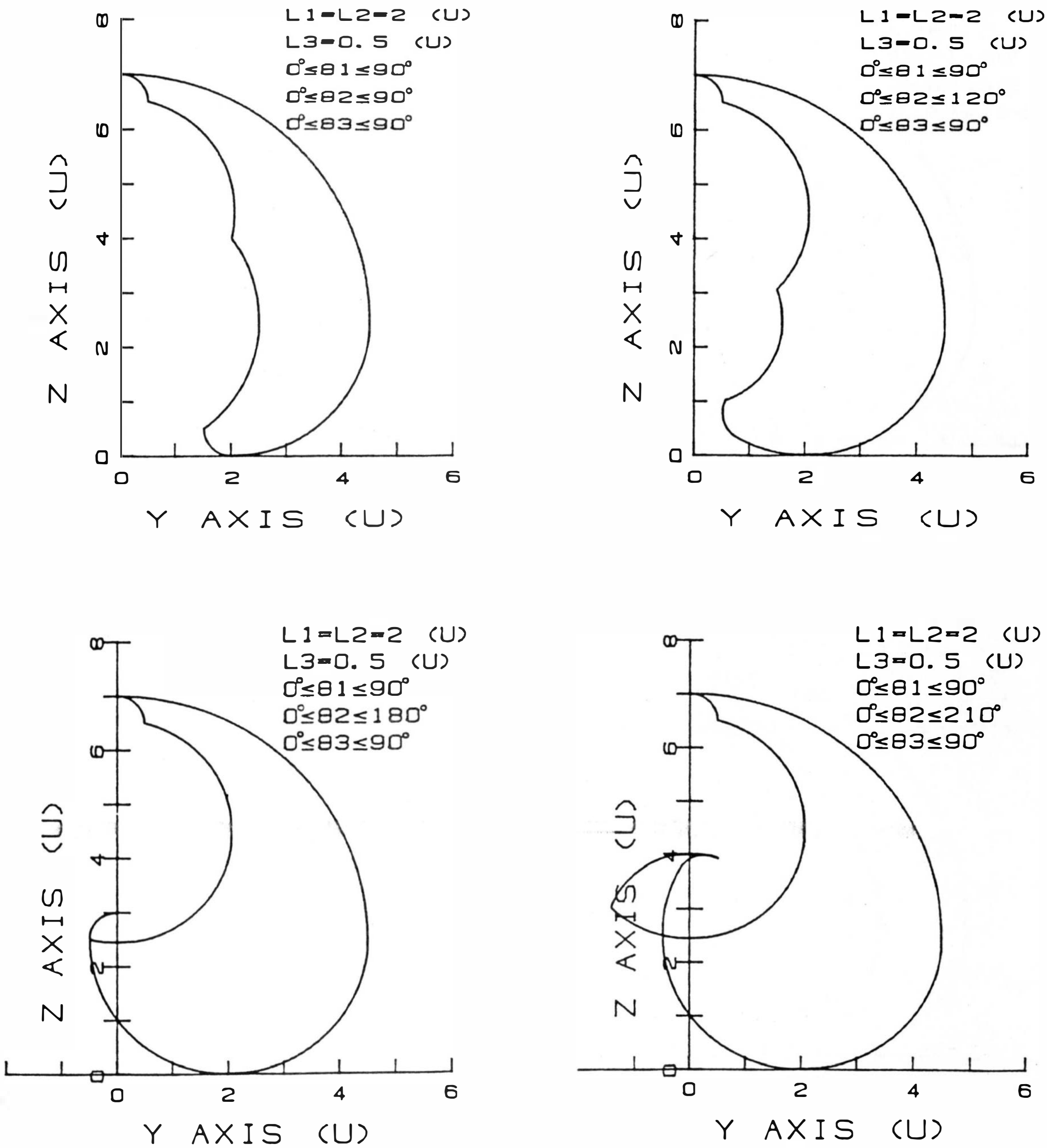


Figure 23. Effect of Joint Displacement (due to θ2)

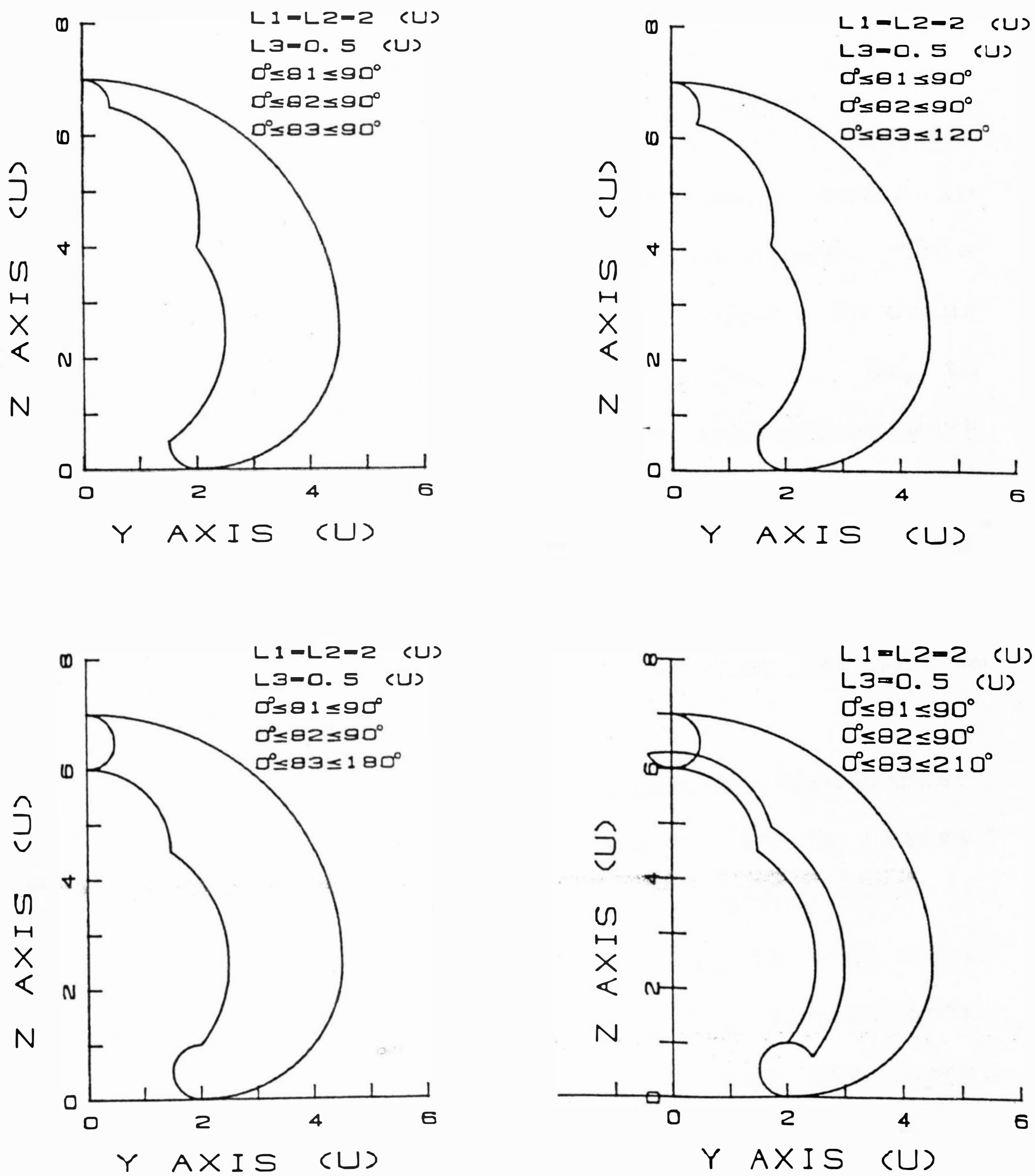


Figure 24. Effect of Joint Displacement (due to θ3)

Chapter IV

THE COMMON WORK SPACE OF TWO ROBOT ARMS

Two robots should communicate with each other in order to work together within a common working space. This requires that the two robots have high intelligence in order to work together effectively, and avoid accidents. Or, to work individually without collisions within the common work space.

To employ two robots on the production line, or any other job, one should know the characteristics of the common work space of two robots in order to set up that job and to control it.

This analysis is conducted to study the boundaries, areas, and volumes of the common work space. It is assumed that the two robots are used in the same assembly line, or work area, with their bases fixed near enough to each other to create a common work space, and that they are oriented facing each other. If a part is brought into the common work space, then both arms will be capable of working interactively with the part. An alternative is to study individual arm actions in the common work space without interference. That is, when one robot's hand is placed ithin the common space, the other robot hand is withdrawn o avoid collisions.

The common work space itself is analyzed, and the oint angles required to move the robot arms to the desired estinations in these conditions are investigated.

1 COMMON WORK SPACE OF TWO-LINK ROBOT ARMS

In Chapter three, we defined the end coordinates of two-link robot arm, and developed an algorithm which alculates the work space. Now, we define the kinematic quations of two robots with respect to the same reference oordinates in order to analyze the common work space of two obots.

As mentioned in Chapter three, to analyze the work ace of an industrial robot having six degrees of freedom, e can treat the gripper as a point, a line, or a rigid dy. Analyzing the regional structures, one can find the aracteristics of the work space of a robot. The analysis the common work space of two robots is based on the llowing assumptions:

1. the bases of two robots are on the same elevation
2. the grippers (hands) are treated as points
3. the two robots are situated facing each other.

4.1.1 Kinematic Equations

The end coordinates of the two robot arms have to be expressed with respect to the same reference coordinates to analyze their interactions. In order to accomplish this, one can express the end-point coordinates of the first robot (left-hand side) from Figure 25 and Equation (3-7) as:

$$
\begin{aligned}
XL &= (L1\sin(\theta 1L) + L2\sin(\theta 1L+\theta 2L))\cos(\gamma L) \\
YL &= (L1\sin(\theta 1L) + L2\sin(\theta 1L+\theta 2L))\sin(\gamma L) \qquad (4\text{-}1) \\
ZL &= L1\cos(\theta 1L) + L2\cos(\theta 1L+\theta 2L) + d
\end{aligned}
$$

Because two robots are involved, the following notations will be used to distinguish the kinematic equations for each robot:

1. XL, YL, ZL in equation (4-1) are the end point coordinates of the first robot (L indicates the left-hand robot)
2. XR, YR, ZR in equation (4-7) are the end point coordinates of the second robot (R indicates the right-hand robot)
3. θ1L is the rotation angle of the first link of the left robot
4. θ2L is the rotation angle of the second link of the left robot

5. γL is the rotation angle of the base of the left robot
6. $\theta 1R$ is the rotation angle of the first link of the right robot
7. $\theta 2R$ is the rotation angle of the second link of the right robot
8. γR is the rotation angle of the base of the right robot

The end point coordinates of the right robot with respect to its base coordinate system can be obtained by finding Ai matrices for each joint and the T2 transformation matrix from Ai matrices as follows:

$$Ai = \text{Rot } Z(\gamma)\ \text{Trans}(0,0,E)\ \text{Trans}(0,R,0)\ \text{Rot } X(\alpha)$$

$$= \begin{bmatrix} \cos(\gamma) & \sin(\gamma)\cos(\alpha) & -\sin(\gamma)\sin(\alpha) & R\sin(\gamma) \\ -\sin(\gamma) & \cos(\gamma)\cos(\alpha) & -\cos(\gamma)\sin(\alpha) & R\cos(\gamma) \\ 0 & \sin(\alpha) & \cos(\alpha) & E \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4\text{-}2)$$

The difference is base rotation between the general Ai matrices for the left and right robots. In other words, to have a common work space, the base rotation of the right robot should be the direction opposite that of the base rotation of the left robot.

From the Ai matrix of Equarion (4-2), A0, A1, and A2 matrices for each joint can be obtained

$$A0 = \begin{bmatrix} \cos(\gamma R) & 0 & -\sin(\gamma R) & 0 \\ -\sin(\gamma R) & 0 & -\cos(\gamma R) & 0 \\ 0 & 1 & 0 & E \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4-3}$$

$$A1 = \begin{bmatrix} \cos(\theta 1R) & \sin(\theta 1R) & 0 & R1\sin(\theta 1R) \\ -\sin(\theta 1R) & \cos(\theta 1R) & 0 & R1\cos(\theta 1R) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4-4}$$

$$A2 = \begin{bmatrix} \cos(\theta 2R) & \sin(\theta 2R) & 0 & R2\sin(\theta 2R) \\ -\sin(\theta 2R) & \cos(\theta 2R) & 0 & R2\cos(\theta 2R) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4-5}$$

Therefore, the T2 matrix for the right robot can be obtained by taking the product of the Ai (i=0, 1, 2) matrices as done in Section 3.1.

$$T2 = A0 \cdot A1 \cdot A2$$

$$= \begin{bmatrix} \cos(\gamma R)\cos(\theta 1R+\theta 2R) & \cos(\gamma R)\sin(\theta 1R+\theta 2R) & -\sin(\gamma R) & \cos(\gamma R)(R1\sin(\theta 1R) + R2\sin(\theta 1R+\theta 2R)) \\ -\sin(\gamma R)\cos(\theta 1R+\theta 2R) & -\sin(\gamma R)\sin(\theta 1R+\theta 2R) & -\cos(\gamma R) & -\sin(\gamma R)(R1\sin(\theta 1R) + R2\sin(\theta 1R+\theta 2R)) \\ -\sin(\theta 1R+\theta 2R) & \cos(\theta 1R+\theta 2R) & 0 & R1\cos(\theta 1R) + R2\cos(\theta 1R+\theta 2R) + E \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4\text{-}6)$$

From Equation (4-6), the end coordinates of the right robot with respect to its base coordinates can be obtained as

$$\begin{aligned} XR &= (R1\sin(\theta 1R) + R2\sin(\theta 1R+\theta 2R)) \cos(\gamma R) \\ YR &= -(R1\sin(\theta 1R) + R2\sin(\theta 1R+\theta 2R)) \sin(\gamma R) \\ ZR &= R1\cos(\theta 1R) + R2\cos(\theta 1R+\theta 2R) + E \end{aligned} \quad (4\text{-}7)$$

Equation (4-7) expresses the end coordinates of the right robot arm with respect to its base coordinate system.

Equation (4-7) can be expressed with respect to the left robot's base coordinate system with the following transformation:

$$XR = XL$$
$$YR = S - YL \qquad (4\text{-}8)$$
$$ZR = ZL$$

in which S is the base distance between the two robots.

Using Equations (4-1) and (4-8), one can write a program to analyze the common work space of the two robot arms.

So far, it is assumed that the center of the common work object coincides with the gripper centers and both grippers are treated as a single point. If the object's size is large compared to the size of the hand, the geometry of the object should be considered.

If the grippers and the object are assumed as a line for considering the geometry of the object, the length of the object between two grippers can be broken down to three components due to its position within the common work space of the two robots. Therefore, Equation (4-8), which shows the coordinate relation of the two robots, will be modified as follows:

$$XR = XL + Qx$$
$$YR = S - (YL + Qy)$$

$$ZR = ZL + Qz$$

in which

Q is the length of an object between two gripper centers

Qx is the X component of Q

Qy is the Y component of Q

Qz is the Z component of Q

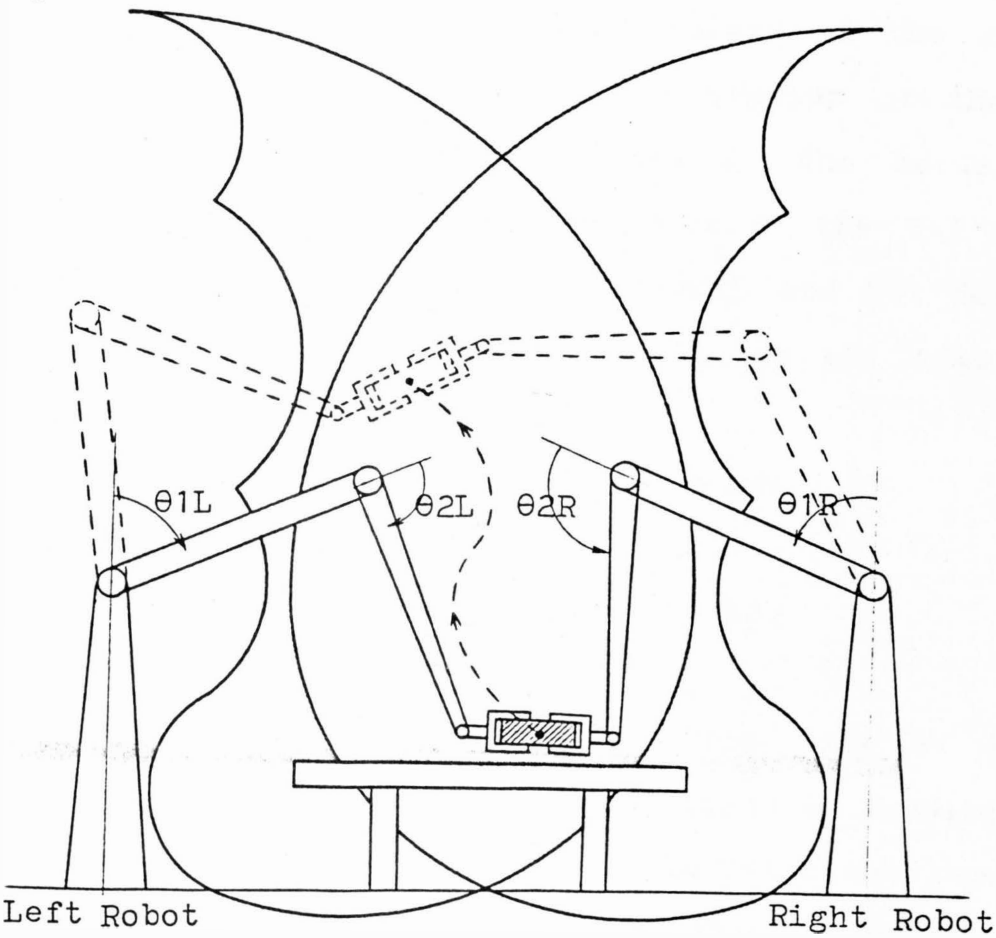


Figure 25. Common Work Space of Two Robots

4.1.2 Boundary and Volume of Common Work Space

Having the end coordinate equations of two robots, it is now possible to find the boundary and volume of the common work space quantitatively. The overlapping of two individual work areas defines the boundary of the common work space. Figure 26 represents this overlap of the two individual work spaces. For a given Z, the horizontal coordinates describing the boundary arcs on the Y-Z plane are Y1i for $\widehat{ABC}$, Y2i for $\widehat{ADC}$, Y3i for $\widehat{A'B'C'}$, and Y4i for $\widehat{A'D'C'}$ (Figure 26). The following four conditions are essential for finding the overlaped area:

1. Y4i > Y1i, Y2i < Y3i and Y2i > Y4i
2. Y4i < Y1i and Y2i < Y3i
3. Y4i < Y1i, Y2i > Y3i and Y3i > Y1i
2. Y4i > Y1i and Y2i > Y3i

In other words, if for a given Z value, one of these conditions is met, then overlap of work space is possible on the Y-Z plane. Knowing the overlap on the Y-Z plane, one can find the overlap on X-Y plane for the given Z value. The boundary arcs on the X-Y plane are YY2i for $\widehat{aa}$, YY1i for $\widehat{bb}$, YY4i for $\widehat{cc}$, and YY3i for $\widehat{dd}$. The same four conditions of overlap mentioned above are also to be met on the X-Y plane.

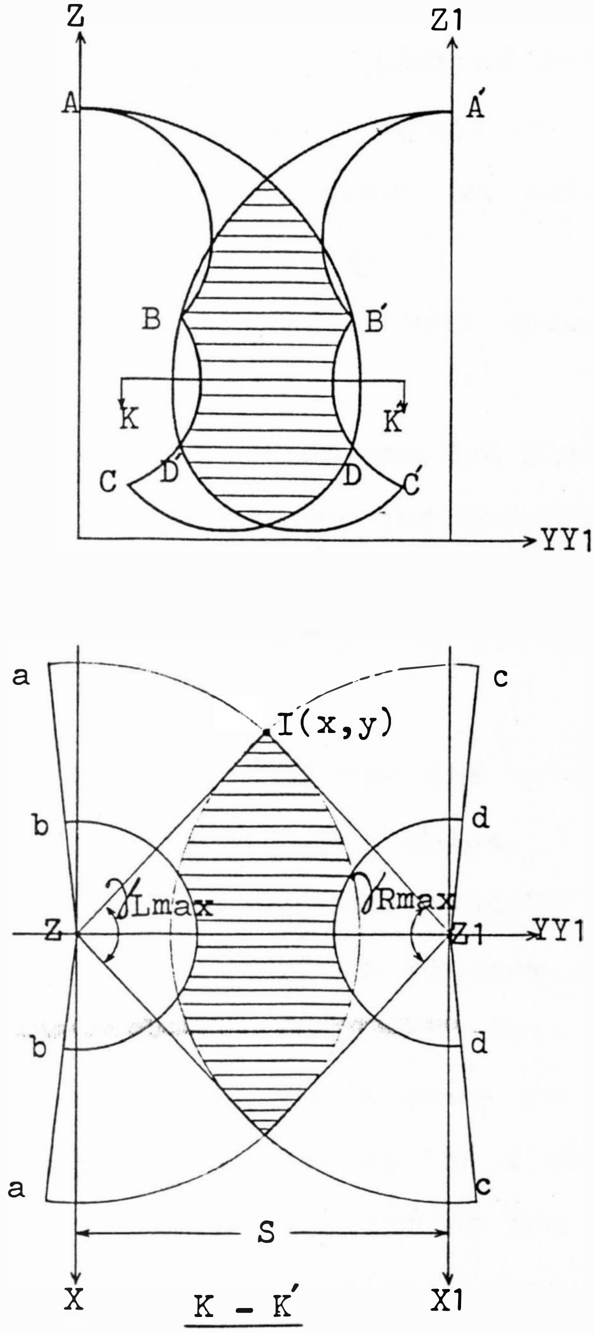


Figure 26: Contour of the Common Work Space of Two Robots

4.1.3 Analysis of the Common Work Space

Knowing the overlap of the work spaces on the Y-Z and X-Y planes, one can calculate the volume of the common work space for given horizontal base distance and arm parameters of the two robots. An algorithm for calculating the common volume is developed as follows:

1. find the boundary of the individual work space on the Y-Z plane
2. find the overlap of work space on the Y-Z plane
3. find the overlap on the X-Y plane for the Z value of part 2
4. calculate the area of overlap on the X-Y plane for the given Z value
5. calculate the volume by multiplying the area on the X-Y plane by the increment of the Z value
6. repeat steps 3 to 5 for the new increment Z.

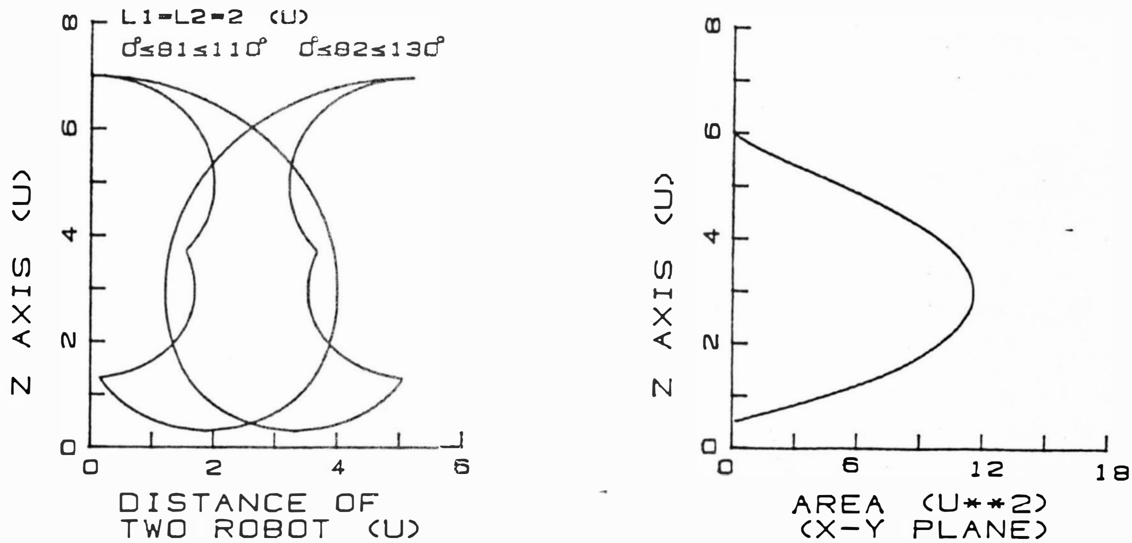
Using this algorithm, a computer program in BASIC was written to illustrate some examples. This program defines the boundaries of the common work space for the two robots and draws the work space boundaries for a given base distance between the two robots. After finding the starting point of the overlap on the Y-Z plane, the program calculates the area on the Y-Z plane and on the X-Y plane, and then calculates the volume of the common work space. Furthermore, the program will search for the maximum height of the overlap on the Y-Z plane and the maximum depth and width on the X-Y plane. The program will execute the above task for variations in the base distance between the two robots.

Figure 27 represents a sample output of the program for the given base distance of the two robots. This example was done assuming that the two robots are identical. From Figure 27, one can get the following informations:

1. the total working area on the Y-Z plane
2. the maximum width and height of the common working area on the Y-Z plane
3. the total common work space volume
4. the common work area on the X-Y plane for a given Z value
5. the maximum depth and width of the working space for a given Z value

where the explanation of glossaries used in Figure 27 is shown in Figure 28.

After executing the sample program for a number of different base distances between the two robots, one can conclude that the volume of the common work space increases as the base distance between the two robots is decreased. The maximum common work space of the two robots will occur when the distance of the two robots is zero.



TOTAL AREA = 8.89 (U**2) (Y-Z PLANE)
MAX. WIDTH = 2.49 (U) (Y-Z PLANE)
MAX. HEIGHT = 5.6 (U) (Y-Z PLANE)
DIS. OF ROBOTS = 5.2 (U) (Y-Z PLANE)
TOTAL VOLUME = 40.92 (U**3)

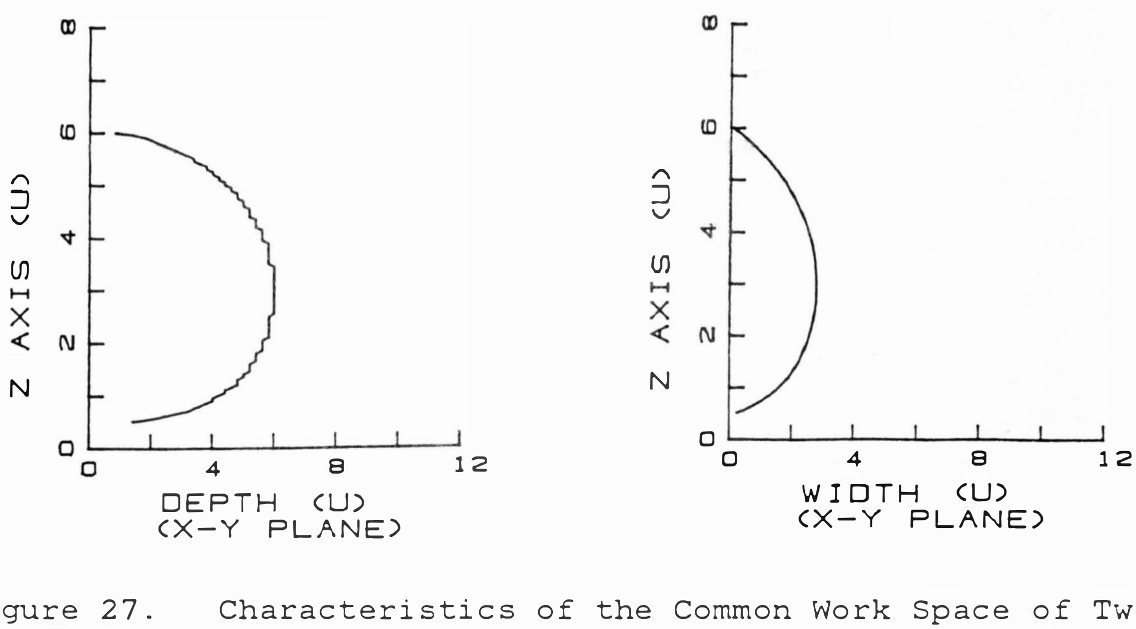


Figure 27. Characteristics of the Common Work Space of Two Two-Link Robots (for a given base distance)

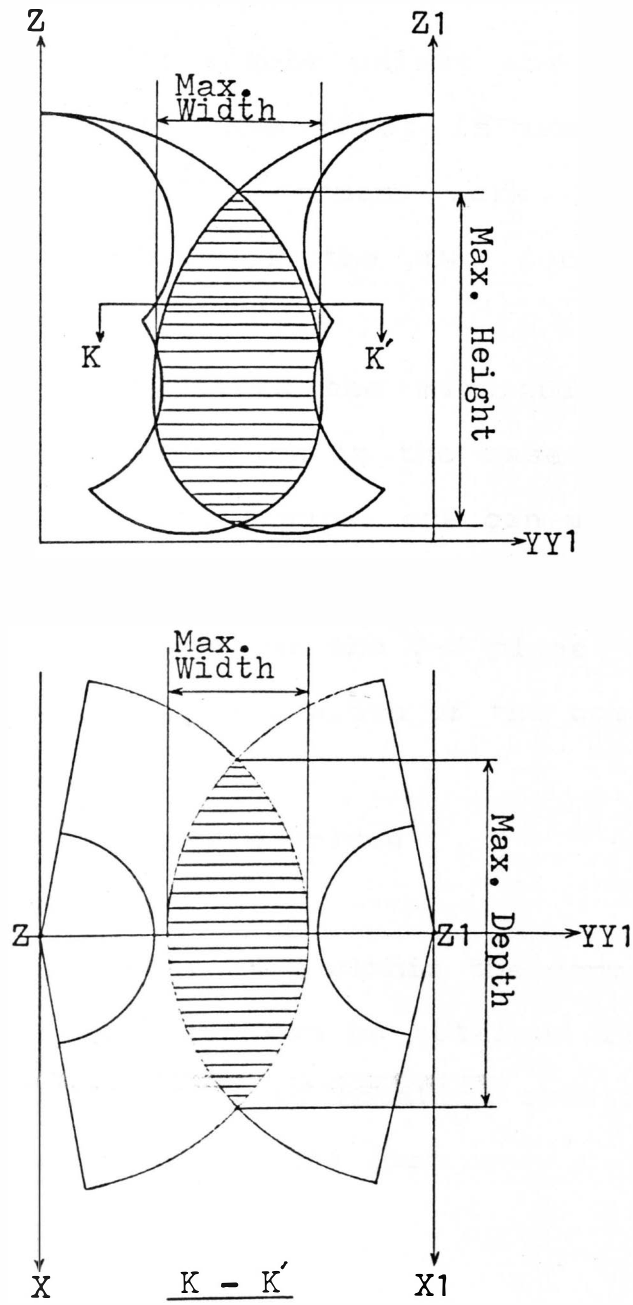


Figure 28: Interpretation of Glossaries in Common Work Space

This is physically impossible unless one robot with both arms steming from the same pivot is used. The physical characteristics of the common work space for the face-to-face orientation of the two robots are further investigated below.

Figure 29 represents the magnitude of the common work space due to a variation in the base distance between the two robots. From this figure, one can get the following information for each assumed base distance:

1. the common work area on the Y-Z plane
2. the maximum height and width of the common work space on the Y-Z plane
3. the common work space volume

To keep the robot hand within the common work space, the limit of base rotation can be obtained from Figure 26. The maximum possible angles of rotation should be less than $\gamma Lmax$ and $\gamma Rmax$ which are of the form

$$\gamma Lmax = 2 \arctan(Ix/Iy)$$
$$\gamma Rmax = 2 \arctan(Ix/(S-Iy) \qquad (4\text{-}9)$$

where Ix and Iy are both referenced to the left robot base coordinates, and

1. Ix is the maximum intersecting X value of the common work space on the X-Y plane.
2. Iy is the Y value at Ix
3. S is the base distance between the two robots
4. γLmax is the angle of rotation of the first robot
5. γRmax is the angle of rotation of the second robot.

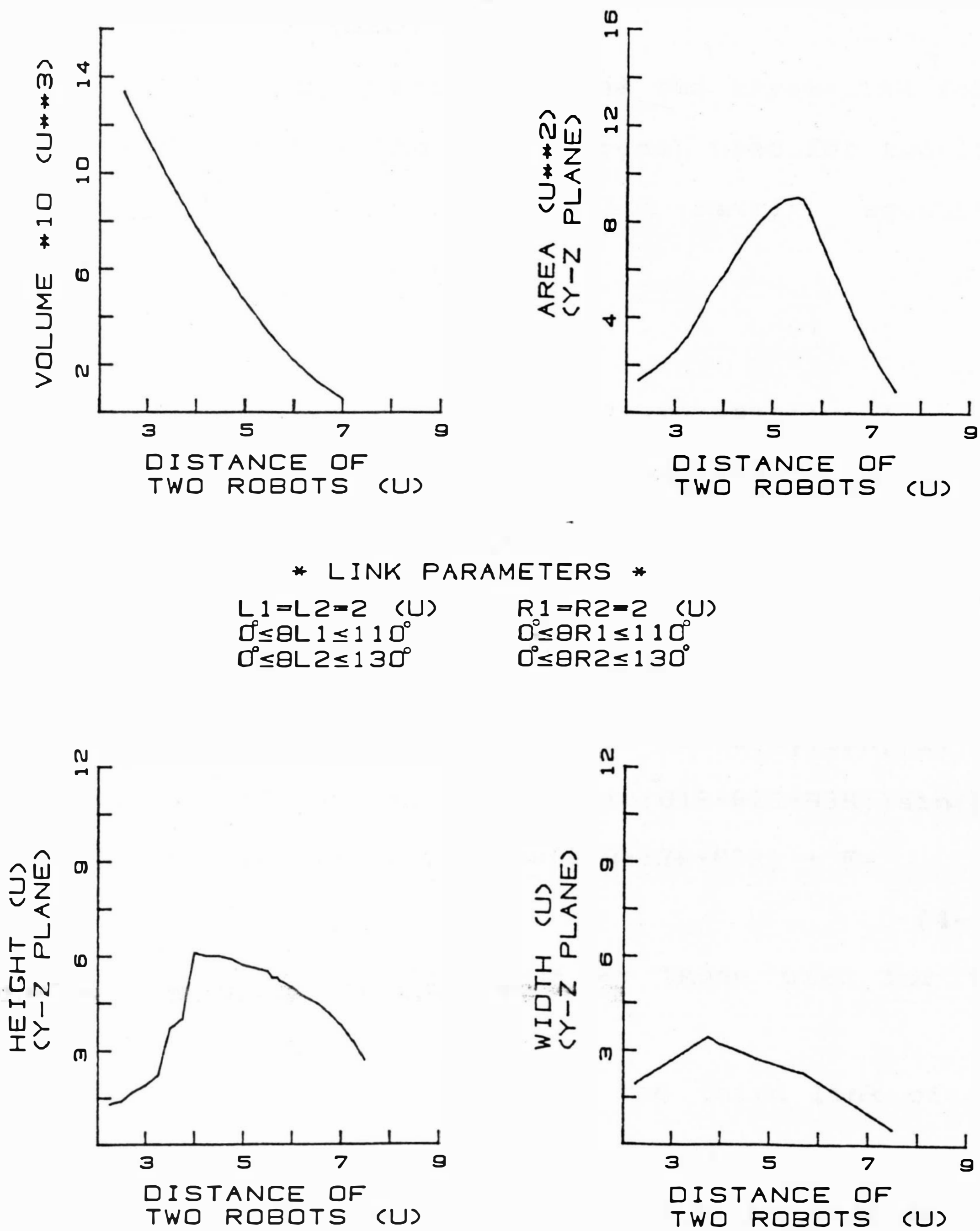


Figure 29. Characteristics of the Common Work Space of Two Two-Link Robots

4.2 TWO THREE-LINK ROBOT ARMS

The kinematic equations of the two three-link robot arms can be obtained by the same approach used for two-link robot arms. From the transformation matrix, equations (4-10) and (4-11) are obtained

$$XL=(L1\sin(\theta 1L)+L2\sin(\theta 1L+\theta 2L)+L3\sin(\theta 1L+\theta 2L+\theta 3L))\cos(\gamma L)$$
$$YL=(L1\sin(\theta 1L)+L2\sin(\theta 1L+\theta 2L)+L3\sin(\theta 1L+\theta 2L+\theta 3L))\sin(\gamma L)$$
$$ZL=L1\cos(\theta 1L)+L2\cos(\theta 1L+\theta 2L)+L3\cos(\theta 1L+\theta 2L+\theta 3L)+d \tag{4-10}$$

$$XR=(R1\sin(\theta 1R)+R2\sin(\theta 1R+\theta 2R)+R3\sin(\theta 1R+\theta 2R+\theta 3R))\cos(\gamma R)$$
$$YR=S-(R1(\sin(\theta 1R)+R2(\sin(\theta 1R+\theta 2R)+R3\sin(\theta 1R+\theta 2R+\theta 3R))\sin(\gamma R)$$
$$ZR=R1\cos(\theta 1R)+R2\cos(\theta 1R+\theta 2R)+R3\cos(\theta 1R+\theta 2R+\theta 3R) + E \tag{4-11}$$

where the notations are the same as those used for the two-link robot arms, and

1. $\theta 3L$ is the rotation angle of the third link of the left robot
2. $\theta 3R$ is the rotation angle of the third link of the right robot.

A computer program was developed in BASIC to produce results similar to those of the case of two-link robot arms.

The common work area on the Y-Z plane increases untill the base distance between the two robots reaches an optimum distance, which gives the maximum common work area on the Y-Z plane. After reaching the optimum distance, the common work area on the Y-Z plane will decrease as the base distance decreases. If the distance of the two robots decreases further, then the common work area on the Y-Z plane will be two parts and, therefore, the main common work space will also be two parts. Figure 30 shows the common work area on the Y-Z and X-Y planes when the two robots are too close. This kind of situation is not desirable and is beyond the scope of this thesis. Furthermore, such cases would be more accurately considered as one robot having two arms with a common shoulder joint rather than two separate robots.

Figures 31 and 32 represent a sample illustration resulted from this program. The interpretation of these figures is the same as in the case of two two-link robot arms (see Figure 27,28,and 29). Using these informations, one can set the robots to get proper working space according to the tasks planned within the common work space. For instance, If the task requires up and down movement, rather than horizontal movement, one can find an appropriate base distance in Figures 28 and 31 to get the proper height of the work space on the Y-Z plane. For a horizontal movement task, one can also choose the appropriate base distance of the two robots from the results of these analyses. With this information available, one can place robots and plan common tasks to get better use of the robots and higher efficiency performing these tasks.

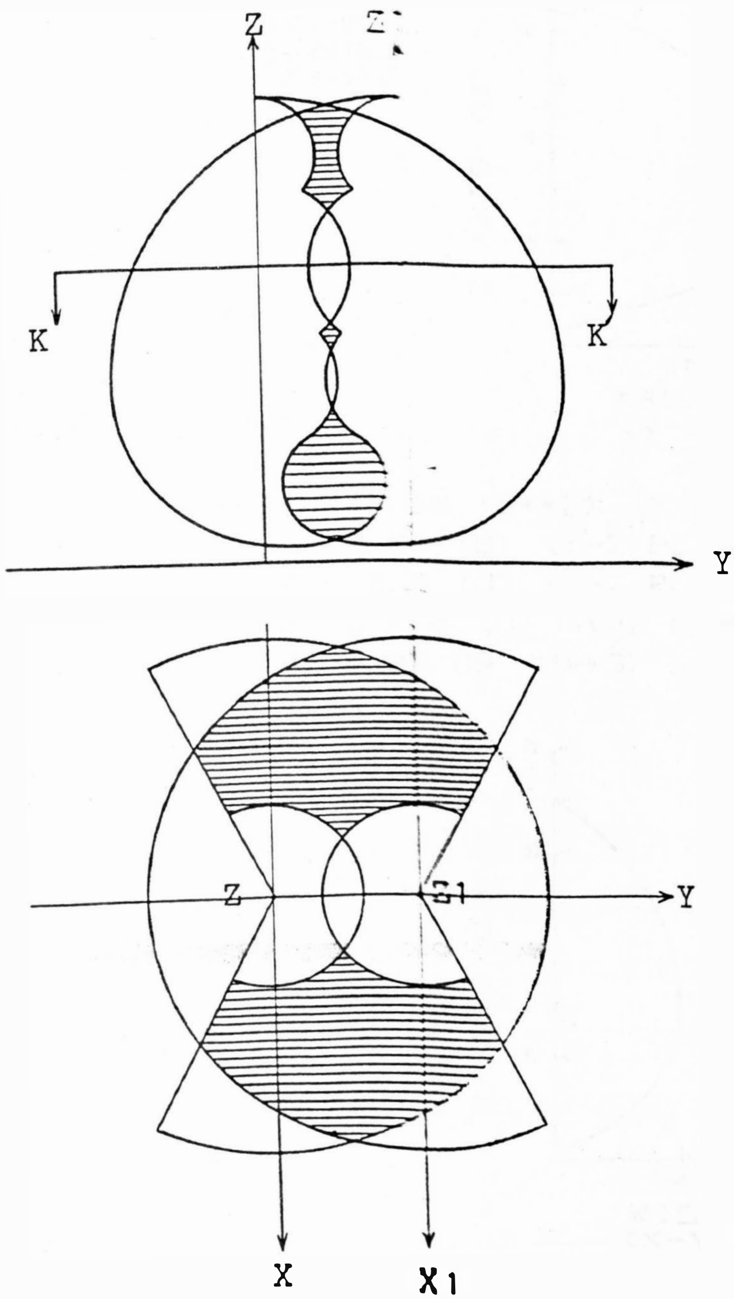
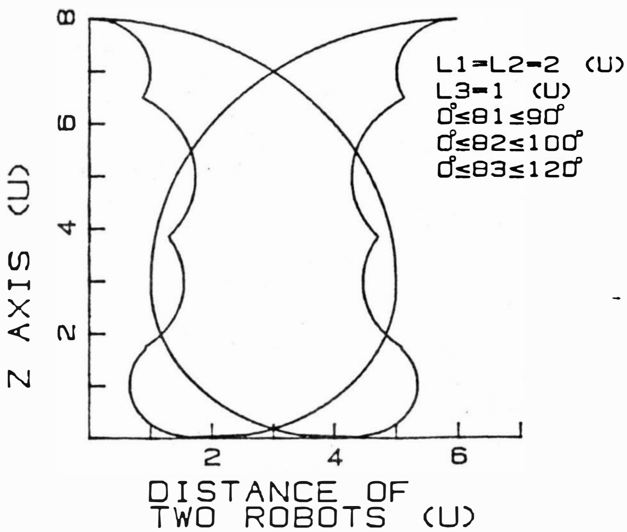
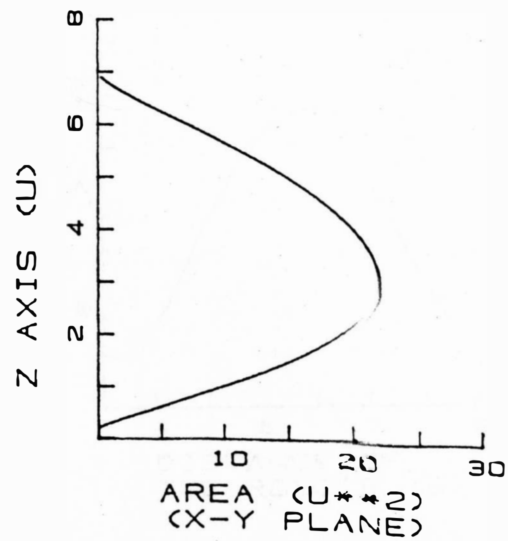
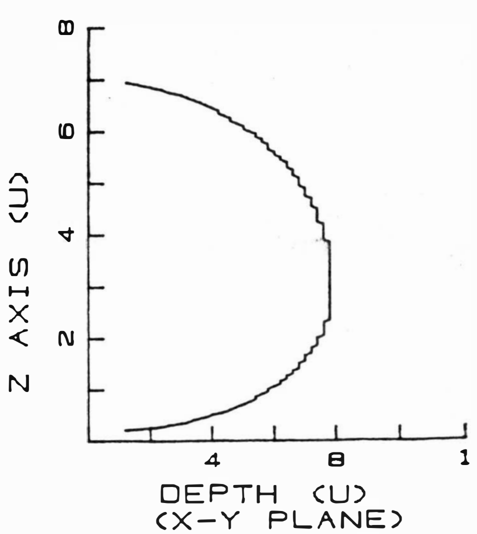


Figure 30. The Shape of Common Work Space on the Y-Z and X-Z Planes





TOTAL AREA = 17.16 (U**2) (Y-Z PLANE)
MAX. WIDTH = 3.66 (U) (Y-Z PLANE)
MAX. HEIGHT = 6.9 (U) (Y-Z PLANE)
DIS. OF ROBOTS = 6 (U) (Y-Z PLANE)
TOTAL VOLUME = 93.09 (U**3)



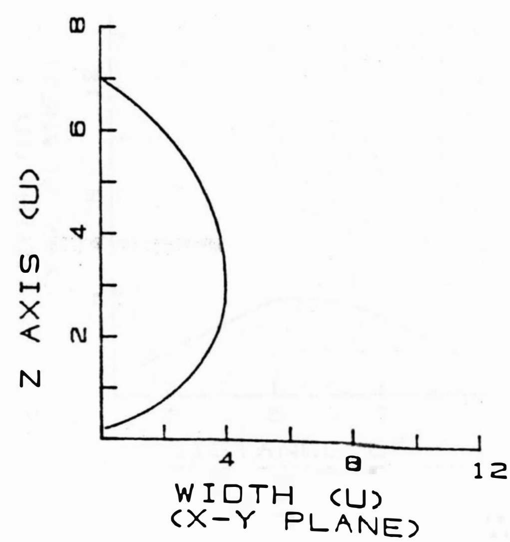


Figure 31. Characteristics of the Common Work Space of Two Three-Link Robots (for a given base distance)

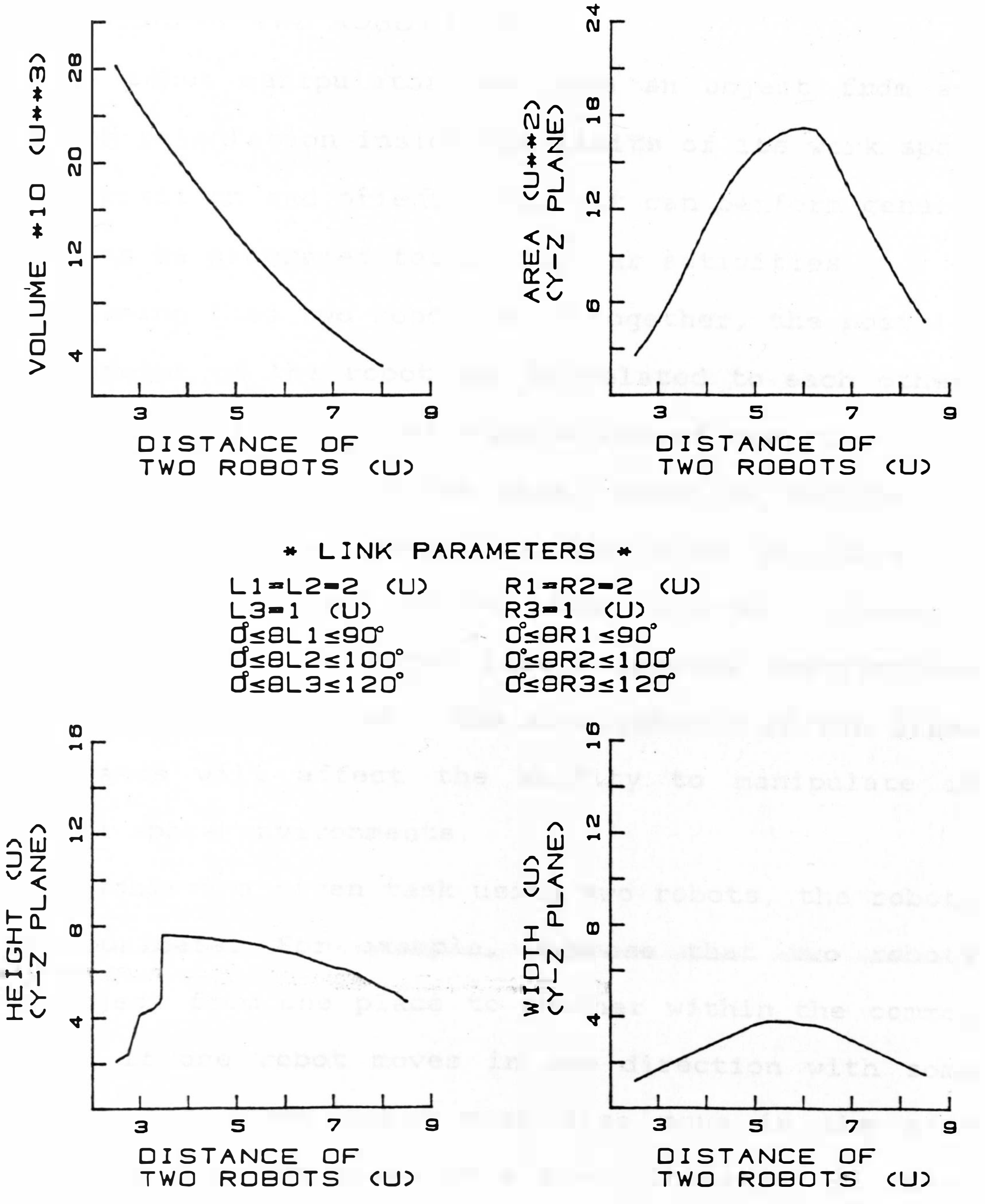


Figure 32. Characteristics of the Common Work Space of Two Three-Link Robots

4.3 SYNTHESIS OF TWO ROBOT ARMS

The robot manipulator can move an object from any position and orientation inside the limits of its work space to another position and orientation. It can perform general tasks and can be programed for particular activities.

Assuming that two robots work together, the position of the end point of the robot arm is related to each other. According to the position and orientation of one robot, the position and orientation of the other robot is defined. To find the joint angle of each link for given position and orientation, one should solve the inverse kinematic equations. However, the robot itself imposes restrictions on the tasks it can perform. The arrangements of the links and joint axes will affect the ability to manipulate in complex work space environments.

To achieve a given task using two robots, the robots should communicate. For example, suppose that two robots carry an object from one place to another within the common work space. If one robot moves in one direction with some speed, then, the other robot must also move in the same direction with the same speed at a given instant. If these conditions are not met, undesirable incidents, such as dropping, squeezing, deformation, or misorientation of the object, may take place. Therefore, the control of two robots working together within a common work space will be a difficult task.

Chapter V

RECOMMENDATIONS

This study included the analysis of the characteristics of the common work space of two robots by studying the work space of their regional structures.

The next step in pursuing this work should be the study of the common work space of two robots by studying the work space of their orientational structures.

Another extension of this work is to consider that the robots have a combination of prismatic pairs revolute pairs. In order to have the larger common work space, the existence of prismatic pairs will be essential. The most complex task lies in the development of the appropriate mathematical tools to study the behaviour of such work space rigorously.

Chapter VI

CONCLUSIONS

The main objective of the present study was to analyze the characteristics of the common work space of two robots. To achieve this objective, the kinematic equations of two-link and three-link robot arms were derived using the 4 x 4 homogeneous transformation matrices. Using these kinematic equations, the boundaries of the work space were defined and the effects of the link parameters were studied.

An algorithm was developed to find the common work space of two robots, and to calculate the work space volume and the work space area on the Y-Z plane. The computer programs are written in BASIC to do those studies based on the kinematic equations derived for the two-link and three-link robot arms.

The following observations have been made in the course of conducting this study:

1. For two-link robot arms
   a) the work space on the Y-Z plane will be maximum when the ratio of L2 and L1 is one (Figure 13)
   b) the work area on the Y-Z plane increases as the joint limits of the second link increases. After

reaching 180 degrees, an extra work space is created (see Figure 14)

c) the work area on the Y-Z plane increases as the joint limit of the first link increases. The shape of the work space becomes circular when the joint limit is 360 degrees.

2. For three-link robot arms

a) the link length ratio of L3 and L giving maximum area on the Y-Z plane becomes smaller as the joint limits increase (Figure 19 and Table 3-1)

b) the link length ratio of L2 and L1 at a fixed L3 giving the maximum area on the Y-Z plane becomes larger as the joint limits increase (Figure 20 and Table 3-2)

c) the ratio of L3 and L2 giving the maximum area on the Y-Z plane becomes smaller as the joint limits become larger (Figure 21 and Table 3-3)

d) the second and the third link will create the extra work space when the displacement exceeds 180 degrees

e) the effects of changing the first link displacement will be the same as the result of two-link robot arms

3. For the common work space

a) the volume of the common work space increases as the base distance between the two robots decreases (Figure 31 and 32)

b) the maximum work space of two robots will occur when the base distance of the two robots is zero

c) the maximum common work area on the Y-Z plane will occur at an optimum base distance (Figure 31 and 32)

d) the common work space will be two parts when the two robots are too close to each other (Figure 30)

It is believed that this study is the first in the analysis of the characteristics of the common work space of two robots. The material presented here will be useful to those who have an interest in the common work space of several robots and in the control of two robots within the common work space interactively or individually.

REFERENCES

1. Roth B. Performance Evaluation of Manipulators from a Kinematic View Point, NBS Special Publication No. 459, Performance Evaluation of Programmable Robots and Manipulators, 1975, pp.39-61.

2. Gupta,K. C. and Roth B. Design Consideration for Manipulator Workspace. ASME Journal of Mechanical Design, Vol. 104, October 1982, pp. 704-711.

3. Tsai, Y. C. and Soni, A. H. Accessible Region and Synthesis of Robot Arms. ASME, Journal of Mechanical Design, vol. 103, October 1981, pp. 803-811.

4. Tsai, Y. C. and Soni, A. H. The Effect of Link Parameters on the Working Space of General 3R Robot Arms. Applied Mechanisms Conference(7th), December,1981, paper No. 3.

5. Tsai, Y. C. and Soni, A. H. An Algorithm for the Workspace of a General n-R Robot. ASME Journal of Mechanisms, Transmissions, and Automation in Design, Vol. 105, March 1983, pp. 52-57.

6. Kumar, A. and Waldron, K.J. The Workspace of a Mechanical Manipulator. ASME journal of Mechanical Design, Vol. 103, July 1981, pp. 665-672.

7. Yang, D. C. H. and Lee, T. W. On the Work Space of Mechanical Manipulators. ASME Journal of Mechanisms, Transmissions, and Automation in Design, Vol. 105,March 1983, pp. 62-69.

8. Lee, T. W. and Yang, D. C. H. On the Evaluation of Manipulator Work Space. ASME Journal of Mechanisms, Transmissions, and Automation in Design, Vol. 105,March 1983, pp. 70-77.

9. Kohli, D. and Spanos, J. Workspace Analysis of Mechanical Manipulators using Polynomial Discriminants. ASME paper No. 84-DET-121.

10. Spanos, J. and Kohli, D. Workspace Analysis of Regional Structure of Manipulators. ASME paper No. 84-DET-120.

11. Sugimoto, K. and Duffy, J. Determination of Extreme Distance of a Robot Hand - Part 1 General Theory. ASME Journal of Mechanical Design, Vol. 103, July 1981, pp. 631- 636.

12. Sugimoto, K. and Duffy, J. Determination of Extreme Distance of a Robot Hand - Part 2 Robot Arms with Special Geometry. ASME Journal of Mechanical Design, Vol 000.

13. Cwiakala, M. and Lee, T. W. Generation and Evaluation of a Manipulator Workspace based on Optimum Path Search. ASME paper No. 84-DET- 185.

14. Freudenstein and Primrose, E. J. F. On the Analysis and Synthesis of the Workspace of a Three-Link, Turning-Pair Connected Robot Arm. ASME journal of Mechanisms, Transmissions, and Automation in Design, Vol. 106, September 1984, pp. 365-370.

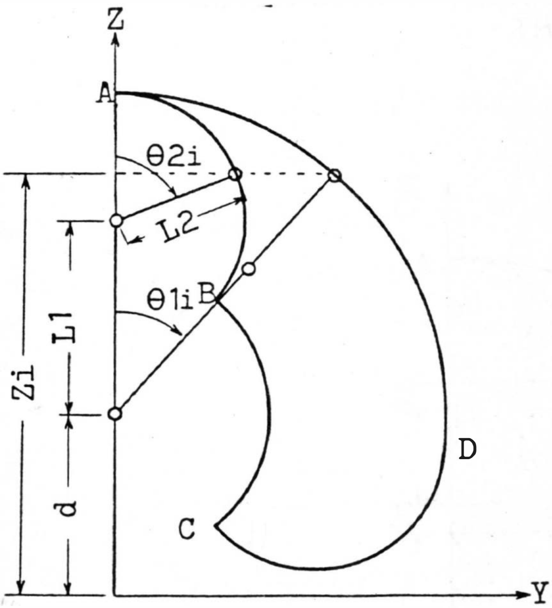
15. Paul,Richard P. Robot Manipulators, Mathematics, Programming, and Control. MIT Press, Cambridge, Mass., 1981.

16. M, Brady, ed. Robot Motion, Planning and Control. MIT Press, Cambridge, Mass.,1982.

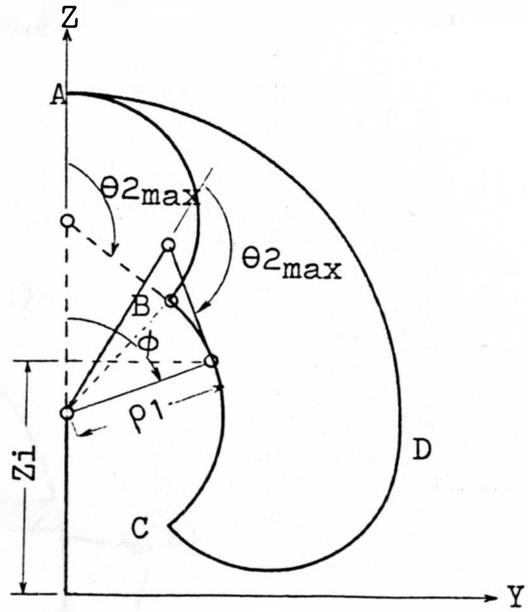
17. Kafrissen E and Stephans M. Industrial Robots and Robotics, Prentice Hall Co., Reston, Virginia, 1984.

Appendix A

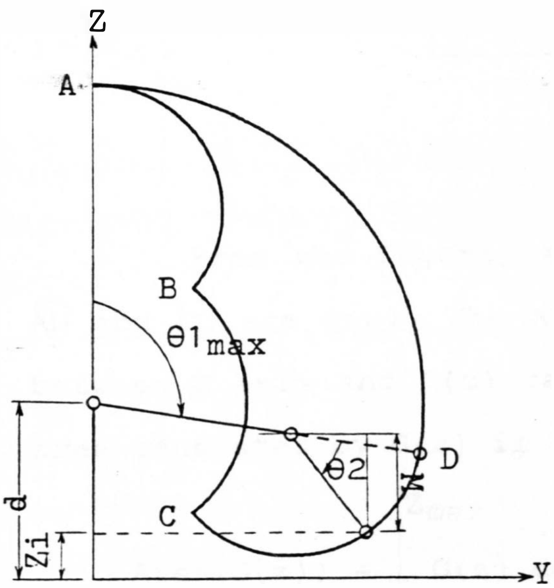
CONFIGURATIONAL SKETCHES



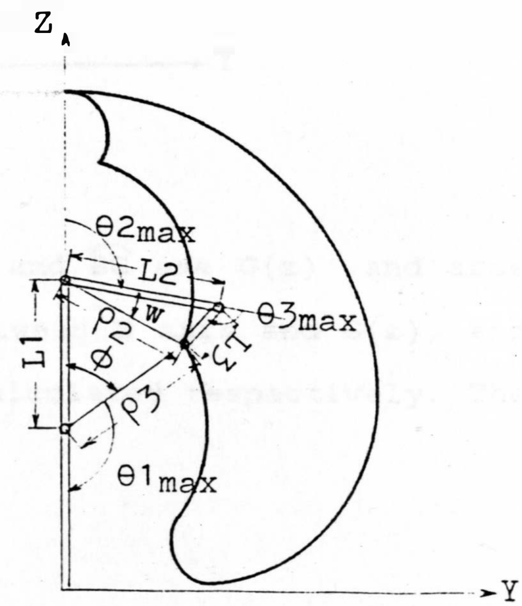
(A-1)



(A-2)



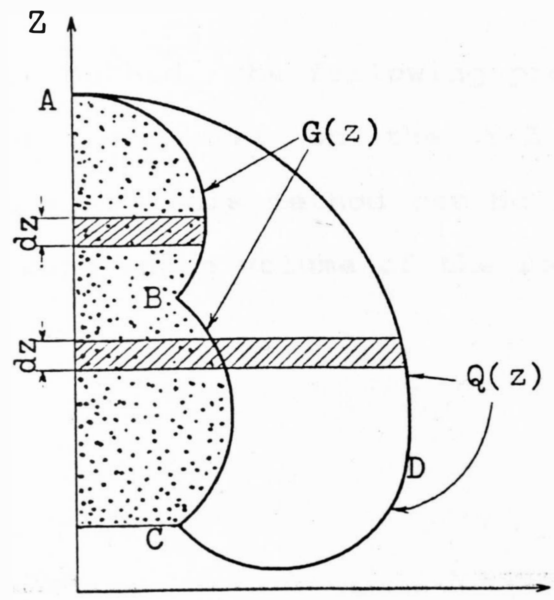
(A-3)



(A-4)

Appendix B

ALTERNATE INTEGRAL METHOD FOR WORK AREA ON THE Y-Z PLANE



From the figure, arcs $\widehat{AB}$ and $\widehat{BC}$ are G(z) ,and arcs $\widehat{AD}$ and $\widehat{DC}$ are Q(z). The areas between Z axis and G(z), and between Z axis and Q(z) can be calculated respectively. The area generated by G(z) is

$$Area(G(z)) = \int_{Z_{min}}^{Z_{max}} G(z)\, dz$$

the area generated by Q(z) is

$$\text{Area}(Q(z)) = \int_{Z_{min}}^{Z_{max}} Q(z)\, dz$$

Therefore, the work space area on the Y-Z plane will be

$$A(y,z) = \text{Area}(Q(z)) - \text{Area}(G(z))$$

Using this method, the following program is written to calculate the work area on the Y-Z plane for the articulated robot arms. This method can be expanded further to calculate the work space volume of the robot arms.

```
 10 ! ** ALTERNATIVE INTEGRAL ME
    THOD **
 20 !
 30 DEG
 40 OPTION BASE 1
 50 DIM Y1(500),Y2(500),Z1(500),
    Z2(500)
 60 L1=2 ! LENGTH OF LINK-1
 70 L2=2 ! LENGTH OF LINK-2
 80 Z0=2.5 ! HEIGHT OF BASE
 90 A0=0 ! MIN. ANGLE OF LINK-1
100 A9=110 ! MAX. ANGLE OF LINK-
    1
110 B0=0 ! MIN. ANGLE OF LINK-2
120 B9=130 ! MAX. ANGLE OF LINK-
    2
130 PRINT "PROGRAM IS STARTED"
140 PRINT
150 K=1
160 FOR I=B0 TO B9 STEP 1
170 A=A0
180 B=I
190 Y1(K)=L1*SIN(A)+L2*SIN(A+B)
200 Z1(K)=L1*COS(A)+L2*COS(A+B)+
    Z0
210 K=K+1
220 NEXT I
230 !
240 FOR I=A0 TO A9 STEP 1
250 A=I
260 B=B9
270 Y1(K)=L1*SIN(A)+L2*SIN(A+B)
280 Z1(K)=L1*COS(A)+L2*COS(A+B)+
    Z0
290 K=K+1
300 NEXT I
310 !
320 J=1
330 FOR I=A0 TO A9 STEP 1
340 A=I
350 B=B0
360 Y2(J)=L1*SIN(A)+L2*SIN(A+B)
370 Z2(J)=L1*COS(A)+L2*COS(A+B)+
    Z0
380 J=J+1
390 NEXT I
400 !
410 FOR I=B0 TO B9 STEP 1
420 A=A9
430 B=I
440 Y2(J)=L1*SIN(A)+L2*SIN(A+B)
450 Z2(J)=L1*COS(A)+L2*COS(A+B)+
    Z0
460 J=J+1
470 NEXT I
480 !
490 A1=0
500 A3=0
510 FOR N1=2 TO K-1
520 D1=ABS(Z1(N1-1)-Z1(N1))
```

```
530 A6=ABS(Y1(N1))*D1
540 IF Z1(N1)>Z1(N1-1) AND Y1(N1
    ))0 THEN 570
550 A1=A1+A6
560 GOTO 580
570 A3=A3+A6
580 NEXT N1
590 A1=A1-A3
600 !
610 A2=0
620 A4=0
630 FOR N2=2 TO J-1
640 D2=ABS(Z2(N2-1)-Z2(N2))
650 A7=ABS(Y2(N2))*D2
660 IF Z2(N2)>Z2(N2-1) AND Y2(N2
    ))0 THEN 690
670 A2=A2+A7
680 GOTO 700
690 A4=A4+A7
700 NEXT N2
710 A2=A2-A4
720 !
730 A=A2-A1
740 PRINT "AREA ON THE Y-Z PLANE
     =";A
750 !
760 GOSUB 910
770 MOVE Y1(1)*3/4,Z1(1)
780 FOR I=2 TO K-1
790 DRAW Y1(I)*3/4,Z1(I)
800 NEXT I
810 !
820 MOVE Y2(1)*3/4,Z2(1)
830 FOR I=2 TO J-1
840 DRAW Y2(I)*3/4,Z2(I)
850 NEXT I
860 END
870 !
880 !
890 ! ** THIS SUBROUTINE IS TO D
    RAW THE AXES **
900 !
910 REM
920 PEN 1 @ GCLEAR
930 SCALE -3,7,-2,8
940 XAXIS 0,.75
950 YAXIS 0,1
960 LDIR 0
970 FOR X=1 TO 10 STEP 2
980 MOVE X*3/4-.1,-.7
990 LABEL VAL$(X)
1000 NEXT X
1010 LDIR 90
1020 FOR Y=1 TO 10 STEP 2
1030 MOVE -.25,Y-.1
1040 LABEL VAL$(Y)
1050 NEXT Y
1060 RETURN
1070 END
```

Appendix C

PROGRAM FOR CALCULATING THE WORK SPACE OF A TWO-LINK ROBOT ARM

This program is to calculate the work area on the Y-Z plane and to draw the contour of the work space on the Y-Z plane.

```
10  ! ** THIS PROGRAM IS TO CALC
    ULATE THE WORK AREA ON THE Y
    -Z PLANE,
20  ! AND TO DRAW THE CONTOUR OF
     THE WORK SPACE ON THE Y-Z P
    LANE **
30  DEG
40  OPTION BASE 1
50  DIM Z(500),Y1(500),Y2(500)
60  PRINT "PROG IS STARTED"
70  PRINT
80  PRINT
90  Z1=.1  ! INCREMENT OF Y
100 Z0=3 ! HEIGHT OF BASE
110 B1=110 ! MAX. ANG. OF LINK-1
120 B2=130 ! MAX. ANG. OF LINK-2
130 L1=2 ! LENGTH OF LINK-1
140 L2=2 ! LENGTH OF LINK-2
150 !
160 ! A1 IS THE CURRENT ANGLE OF
     L1
170 ! A2 IS THE CURRENT ANGLE OF
     L2
180 !
```

```
200 Z(1)=L1+L2
210 Y1(1)=0
220 Y2(1)=0
230 !
240 FOR K=2 TO 300 STEP 1
250 Z(K)=Z(K-1)-Z1
260 IF Z(K)<-(-(L1*COS(B1))+L2)
    THEN 310
270 NEXT K
280 !
290 !
300 !
310 A2=0
320 FOR I=2 TO K
330 A1=0
340 IF A2>B2 THEN 370
350 A2=ACS((Z(I)-L1)/L2)
360 IF A2>B2 THEN 370 ELSE 450
370 L=SQR(L1^2+L2^2+2*L1*L2*COS(
    B2))
380 E=ACS((L1^2+L^2-L2^2)/(2*L1*
    L))
390 IF ABS(Z(I))>L THEN 510
400 C=ACS(Z(I)/L)
410 T=C-E
420 IF T>B1 THEN 510
430 Y1(I)=L*SIN(C)
440 GOTO 460
450 Y1(I)=L1*SIN(A1)+L2*SIN(A1+A
    2)
460 N=I
470 NEXT I
480 !
490 !
500 !
510 A1=0
520 FOR J=2 TO K
530 A2=0
540 IF A1>B1 THEN 570
550 A1=ACS(Z(J)/(L1+L2))
560 IF A1>B1 THEN 570 ELSE 680
570 A1=B1
580 IF B1+B2<180 THEN 610
590 Z=ABS(Z(J))-ABS(L1*COS(B1))
600 IF Z<L2 THEN 610 ELSE 710
610 M=Z(J)-L1*COS(B1)
620 Y2(J)=L1*SIN(B1)+SQR(L2^2-M^
    2)
630 IF J>N THEN Y1(J)=2*L1*SIN(B
    1)-Y2(J)
640 Q=(Y2(J)^2+Z(J)^2-L1^2-L2^2)
    /(-(2*L1*L2))
650 A2=180-ACS(Q)
660 IF A2>B2 THEN 710
670 GOTO 690
680 Y2(J)=L1*SIN(A1)+L2*SIN(A1+A
    2)
```

```
690 NEXT J
700 !
710 IF B1+B2<180 THEN 740
720 Y1(J)=L1*SIN(B1)
730 Y2(J)=L1*SIN(B1)
740 !
750 A=0 ! AREA ON THE Y-Z PLANE
760 FOR I=1 TO J
770 D=Y2(I)-Y1(I)
780 A0=D*Z1
790 A=A+A0
800 NEXT I
810 !
820 PRINT "AREA ON THE Y-Z PLANE
     =";A
830 GOSUB 880
835 !
836 END
840 !
845 !
850 !
860 ! ** THIS SUBROUTINE IS TO D
    RAW THE WORK SPACE ON THE Y-
    Z PLANE **
870 !
880 REM
890 PEN 1 @ GCLEAR
900 SCALE -3,7,-2,8
910 XAXIS 0,.75
920 YAXIS 0,1
930 !
940 LDIR 0
950 FOR X=1 TO 8 STEP 2
960 MOVE 3/4*X-.1,-.7
970 LABEL VAL$(X)
980 NEXT X
990 !
1000 LDIR 90
1010 FOR Y=1 TO 10 STEP 2
1020 MOVE -.25,Y-.1
1030 LABEL VAL$(Y)
1040 NEXT Y
1050 !
1060 MOVE Y1(1)*3/4,Z(1)+Z0
1070 FOR I=1 TO J
1080 DRAW Y1(I)*3/4,Z(I)+Z0
1090 NEXT I
1100 !
1110 MOVE Y2(1)*3/4,Z(1)+Z0
1120 FOR I=1 TO J
1130 DRAW Y2(I)*3/4,Z(I)+Z0
1140 NEXT I
1150 COPY
1160 RETURN
1170 END
```

Appendix D

PROGRAM FOR ANALYZING THE COMMON WORK SPACE OF TWO THREE-LINK ROBOTS

This program is to analyze the common work space of two three-link robot arms and outputs the followings:

1. calculates the common work space volume for given base distances between two robots
2. calculates the common work area on the Y-Z plane
3. finds the maximum height of the common work space on the Y-Z plane
4. finds the maximum width of the common work space on the Y-Z plane
5. calculates the common work area on the X-Y plane for given Z values
6. finds the maximum depth of the common work space on the X-Y plane
7. finds the maximum width of the common work space on the X-Y plane

```
10 ! ** THIS PROGRAM IS TO ANAL
   YZE THE COMMON WORK SPACE **
20 !
30 !
40 DEG
50 OPTION BASE 1
60 DIM Z(200),Y1(200),Y2(200),Y
   3(200),Y4(200),S(30),W9(100)
   ,H9(100),A9(100),V9(100)
70 DIM X1(100),X2(100),X3(100),
   X4(100),X9(100),Z9(100),Y9(1
   00),A5(30),H5(30),W5(30)
80 !
90 PRINT " PROGRAM IS STARTED"
100 !
110 Z0=.1 ! INCREMENT OF Z
120 D=3 ! HEIGHT OF BASE
130 B1=90 ! MAX. ANG. OF LINK-1
140 B2=100 ! MAX. ANG. OF LINK-2
150 B3=120 ! MAX. ANG. OF LINK-3
160 L1=2 ! LENGTH OF LINK-1
170 L2=2 ! LENGTH OF LINK-2
180 L3=1 ! LENGTH OF LINK-3
190 Z(1)=L1+L2+L3
200 Y1(1)=0
210 Y2(1)=0
220 !
230 !
240 FOR K=2 TO 200 STEP 1
250 Z(K)=Z(K-1)-Z0
260 IF Z(K)<-(-(L1*COS(B1)+L2*CO
    S(B1+B2))+L3) THEN 290
270 NEXT K
280 !
290 FOR I=2 TO K
300 A1=0
310 A2=0
320 !
330 A3=ACS((Z(I)-L1-L2)/L3)
340 IF A3>B3 THEN 380
350 Y1(I)=L3*SIN(A3)
360 NEXT I
370 !
380 FOR J=I TO K
390 A1=0
400 A3=B3
410 L=SQR(L2^2+L3^2+2*L2*L3*COS(
    B3))
420 E=ACS((L2^2+L^2-L3^2)/(2*L2*
    L))
430 C=ACS((Z(J)-L1)/L)
440 T=C-E
450 IF T>B2 THEN 490
460 Y1(J)=L*SIN(C)
470 NEXT J
480 !
490 H=ACS((L1-Z(J))/L)
500 FOR N=J TO K
510 A2=B2
520 A3=B3
```

```
530 R=SQR(L1^2+L^2-2*L1*L*COS(H)
    )
540 F=ACS((L1+L2*COS(B2)+L3*COS(
    B2+B3))/R)
550 U=L1*COS(B1)+L2*COS(B1+B2)+L
    3*COS(B1+B2+B3)
560 IF Z(N)<U THEN 630
570 G=ACS(Z(N)/R)
580 Q=G-F
590 IF Q>B1 THEN 630
600 Y1(N)=R*SIN(G)
610 NEXT N
620 !
630 FOR I=2 TO K
640 A2=0
650 A3=0
660 A1=ACS(Z(I)/(L1+L2+L3))
670 IF A1>B1 THEN 710
680 Y2(I)=(L1+L2+L3)*SIN(A1)
690 NEXT I
700 !
710 FOR J=I TO K
720 A1=B1
730 A3=0
740 Q1=L1*COS(B1)+L2*COS(B1+B2)-
    L3
750 IF Z(J)<Q1 THEN 830
760 X=SQR((L2+L3)^2-(Z(J)-L1*COS
    (B1))^2)
770 Y2(J)=L1*SIN(B1)+X
780 V=(Y2(J)^2+Z(J)^2-L1^2-(L2+L
    3)^2)/(-(2*L1*(L2+L3)))
790 A2=180-ACS(V)
800 IF A2>B2 THEN 830
810 NEXT J
820 !
830 D=0
840 FOR M=J TO K
850 A1=B1
860 A2=B2
870 IF Z(M)<Q1 THEN 920
880 Z=SQR(L3^2-(Z(M)-(L1*COS(B1)
    +L2*COS(B1+B2)))^2)
890 Y2(M)=L1*SIN(B1)+L2*SIN(B1+B
    2)+Z
900 NEXT M
910 !
920 FOR P=N TO K
930 Q1=L1*COS(B1)+L2*COS(B1+B2)-
    L3
940 IF Z(P)<Q1 THEN 990
950 Z1=SQR(L3^2-(Z(P)-(L1*COS(B1
    )+L2*COS(B1+B2)))^2)
960 Y1(P)=L1*SIN(B1)+L2*SIN(B1+B
    2)-Z1
970 NEXT P
980 !
990 Y1(P)=L1*SIN(B1)+L2*SIN(B1+B
    2)
1000 Y2(M)=Y1(P)
```

```
1010 !
1020 !
1030 S0=.5 ! INCREMENT OF S
1040 S(1)=3 ! DISTANCE OF TWO RO
     BOTS
1050 FOR I1=2 TO 100
1060 S(I1)=S(I1-1)+S0
1070 IF S(I1)>8 THEN 2940
1080 !
1090 FOR I=1 TO M
1100 Y3(I)=S(I1)-Y1(I)
1110 Y4(I)=S(I1)-Y2(I)
1120 NEXT I
1130 !
1140 !
1150 ! ** THIS IS TO DRAW THE CO
     MMON WORK SPACE ON THE Y-Z
     PLANE **
1160 !
1170 PEN 1 @ GCLEAR
1180 SCALE -3,7,-2,8
1190 XAXIS 0,.75
1200 YAXIS 0,1
1210 MOVE 0,0
1220 !
1230 LDIR 0
1240 FOR X=2 TO 10 STEP 2
1250 MOVE X*3/4-.1,-.7
1260 LABEL VAL$(X)
1270 NEXT X
1280 !
1290 LDIR 90
1300 FOR Y=2 TO 10 STEP 2
1310 MOVE -.25,Y-.1
1320 LABEL VAL$(Y)
1330 NEXT Y
1340 !
1350 MOVE Y1(1)*3/4,Z(1)+D
1360 FOR I=1 TO F
1370 DRAW Y1(I)*3/4,Z(I)+D
1380 NEXT I
1390 MOVE Y2(1)*3/4,Z(1)+D
1400 FOR I=1 TO M
1410 DRAW Y2(I)*3/4,Z(I)+D
1420 NEXT I
1430 MOVE Y3(1)*3/4,Z(1)+D
1440 FOR I=1 TO F
1450 DRAW Y3(I)*3/4,Z(I)+D
1460 NEXT I
1470 MOVE Y4(1)*3/4,Z(1)+D
1480 FOR I=1 TO M
1490 DRAW Y4(I)*3/4,Z(I)+D
1500 NEXT I
1510 COPY
1520 PRINT
1530 !
1540 !
1550 ! ** THIS IS TO FIND THE OV
     ERLAP OF TWO WORK SPACES ON
      THE Y-Z PLANE
```

```
1560 ! AND TO CALCULATE THE AREA
       ON THE Y-Z PLANE **
1570 !
1580 D1=0
1590 L=0
1600 W=0
1610 A0=0
1620 FOR I=1 TO M
1630 IF Y4(I)>Y1(I) AND Y2(I)<Y3
     (I) THEN D1=Y2(I)-Y4(I)
1640 IF Y4(I)<Y1(I) AND Y2(I)<Y3
     (I) THEN D1=Y2(I)-Y1(I)
1650 IF Y4(I)<Y1(I) AND Y2(I)>Y3
     (I) THEN D1=Y3(I)-Y1(I)
1660 IF Y4(I)>Y1(I) AND Y2(I)>Y3
     (I) THEN D1=Y3(I)-Y4(I)
1670 A=D1*Z0
1680 IF A>0 THEN A0=A0+A
1690 IF W<D1 THEN W=D1
1700 IF L=1 THEN 1740
1710 IF D1>0 THEN P1=Z(I)
1720 IF D1>0 THEN L=1
1730 GOTO 1760
1740 IF D1<=0 THEN P2=Z(I)
1750 IF D1<=0 THEN L=0
1760 NEXT I
1770 !
1780 H=P1-P2
1790 PRINT "TOTAL AREA ON THE Y-
     Z PLANE =";A0
1800 PRINT "MAX. WIDTH ON THE Y-
     Z PLANE =";W
1810 PRINT "MAX. HEIGHT ON THE Y
     -Z PLANE =";H
1820 PRINT "DISTANCE OF TWO ROBO
     TS =";S(I1)
1830 PRINT
1840 PRINT
1850 !
1860 A5(I1)=A0
1870 W5(I1)=W
1880 H5(I1)=H
1890 N1=I1
1900 !
1910 M1=0
1920 M9=0
1930 V0=0
1940 !
1950 FOR J9=1 TO P
1960 IF Y4(J9)>=Y2(J9) THEN 2650
1970 M1=M1+1
1980 Z9(M1)=Z(J9)+D
1990 !
2000 G1=0
2010 G2=0
2020 G3=0
2030 G4=0
2040 !
2050 X1(1)=Y1(J9)
2060 X2(1)=Y2(J9)
```

```
2070 X3(1)=Y3(J9)
2080 X4(1)=Y4(J9)
2090 X9(1)=0
2100 !
2110 FOR J8=2 TO 150
2120 X9(J8)=J8/10
2130 IF G2>82 THEN 2210
2140 G1=X9(J8)/Y1(J9)
2150 IF G1>1 THEN 2210
2160 G2=ASN(G1)
2170 !
2180 J1=J8
2190 X1(J8)=SQR(Y1(J9)^2-X9(J8)^2)
2200 X3(J8)=S(I1)-X1(J8)
2210 IF G4>82 THEN 2290
2220 G3=X9(J8)/Y2(J9)
2230 IF G3>1 THEN 2290
2240 G4=ASN(G3)
2250 X2(J8)=SQR(Y2(J9)^2-X9(J8)^2)
2260 X4(J8)=S(I1)-X2(J8)
2270 J2=J8
2280 NEXT J8
2290 !
2300 !
2310 IF S(I1)<5.1 OR S(I1)>7.1 THEN 2370
2320 IF M1>M9 THEN GOSUB 4900
2330 !
2340 !
2350 ! ** THIS IS TO CALCULATE THE AREA ON THE X-Y PLANE FOR A GIVEN Z VALUE **
2360 !
2370 X0=.1
2380 D2=0
2390 L=0
2400 W=0
2410 A0=0
2420 P1=0
2430 !
2440 FOR I=1 TO J2
2450 IF I>J1 THEN 2500
2460 IF X4(I)>X1(I) AND X2(I)<X3(I) THEN D2=X2(I)-X4(I)
2470 IF X4(I)<X1(I) AND X2(I)<X3(I) THEN D2=X2(I)-X1(I)
2480 IF X4(I)<X1(I) AND X2(I)>X3(I) THEN D2=X3(I)-X1(I)
2490 IF X4(I)>X1(I) AND X2(I)>X3(I) THEN D2=X3(I)-X4(I)
2500 D2=X2(I)-X4(I)
2510 IF X4(I)>X2(I) THEN 2570
2520 A=D2*X0
2530 IF A>0 THEN A0=A0+A
2540 IF W<D2 THEN W=D2
2550 IF D2>0 THEN P1=X9(I)
2560 NEXT I
2570 !
```

```
2580 !
2590 H=P1
2600 V0=V0+A0*Z0*2
2610 A9(M1)=A0*2
2620 H9(M1)=H*2
2630 W9(M1)=W
2640 !
2650 NEXT J9
2660 !
2670 PRINT "TOTAL VOLUME =";V0
2680 !
2690 V9(I1)=V0
2700 I9=M1
2710 FOR L=1 TO M1
2720 Y9(L)=A9(L)/10
2730 NEXT L
2740 PRINT "AREA ON THE X-Y PLAN
     E VS Z-VALUE"
2750 GOSUB 3940
2760 PRINT
2770 PRINT
2780 FOR L=1 TO M1
2790 Y9(L)=H9(L)/2
2800 NEXT L
2810 PRINT "DEPTH ON THE X-Y PLA
     NE VS Z-VALUE"
2820 PRINT
2830 GOSUB 4270
2840 PRINT
2850 PRINT
2860 FOR L=1 TO M1
2870 Y9(L)=W9(L)/2
2880 NEXT L
2890 PRINT "WIDTH ON THE X-Y PLA
     NE VS Z-VALUE"
2900 PRINT
2910 GOSUB 4270
2920 !
2930 NEXT I1
2940 !
2950 !
2960 FOR K=2 TO N1
2970 Y9(K)=(S(K)-S(1))*3/4
2980 Z9(K)=A5(K)/3
2990 NEXT K
3000 PRINT "AREA ON THE Y-Z PLAN
     E VS DISTANCE OF TWO ROBOTS
     "
3010 GOSUB 3290
3020 !
3030 FOR J=2 TO N1
3040 Z9(J)=W5(J)
3050 NEXT J
3060 PRINT "MAX. WIDTH ON THE Y-
     Z PLANE VS DISTANCE OF TWO
     ROBOTS"
3070 GOSUB 3610
3080 !
3090 FOR L=2 TO N1
3100 Z9(L)=H5(L)
```

```
3110 NEXT L
3120 PRINT "MAX. HEIGHT ON THE Y
     -Z PLANE VS DISTANCE OF TWO
      ROBOTS"
3130 GOSUB 3610
3140 !
3150 !
3160 PRINT "COMMON VOLUME VS DIS
     TANCE OF TWO ROBOTS"
3170 GOSUB 4590
3180 !
3190 !
3200 PRINT
3210 PRINT "THIS IS THE END OF P
     ROGRAM "
3220 END
3230 !
3240 !
3250 !
3260 ! ** THIS SUBROUTINE IS TO
     DRAW THE RELATIONSHIP BETWE
     EN THE AREA ON THE Y-Z PLAN
     E
3270 ! AND THE DISTANCE OF TWO R
     OBOTS **
3280 !
3290 REM
3300 PEN 1 @ GCLEAR
3310 SCALE -3,7,-2,8
3320 XAXIS 0,.75
3330 YAXIS 0,1
3340 !
3350 LDIR 0
3360 FOR X=1 TO 8 STEP 2
3370 MOVE X*3/4-.1,-.7
3380 LABEL VAL$(X+S(1))
3390 NEXT X
3400 !
3410 LDIR 90
3420 FOR Y=2 TO 10 STEP 2
3430 MOVE -.25,Y-.1
3440 LABEL VAL$(Y*3)
3450 NEXT Y
3460 !
3470 MOVE Y9(2),Z9(2)
3480 FOR K=2 TO N1
3490 DRAW Y9(K),Z9(K)
3500 NEXT K
3510 COPY
3520 PRINT
3530 PRINT
3540 RETURN
3550 !
3560 !
3570 !
3580 ! ** THIS SUBROUTINE IS TO
     DRAW THE RELATIONSHIP BETWE
     EN THE HEIGHT OF THE WORK S
     PACE ON
3590 ! THE Y-Z PLANE AND THE DIS
     TANCE OF TWO ROBOTS **
```

```
3600 !
3610 REM
3620 PEN 1 @ GCLEAR
3630 SCALE -3,7,-2,8
3640 XAXIS 0,.75
3650 YAXIS 0,1
3660 !
3670 LDIR 0
3680 FOR X=2 TO 8 STEP 2
3690 MOVE X*3/4-.1,-.7
3700 LABEL VAL$(X+S(1))
3710 NEXT X
3720 !
3730 LDIR 90
3740 FOR Y=2 TO 10 STEP 2
3750 MOVE -.25,Y-.1
3760 LABEL VAL$(Y)
3770 NEXT Y
3780 !
3790 MOVE Y9(2),Z9(2)
3800 FOR K=2 TO N1
3810 DRAW Y9(K),Z9(K)
3820 NEXT K
3830 COPY
3840 PRINT
3850 PRINT
3860 RETURN
3870 !
3880 !
3890 !
3900 !
3910 ! ** THIS SUBROUTINE IS TO
     DRAW THE RELATIONSHIP BETWE
     EN THE AREA ON THE X-Y PLAN
     E
3920 ! AND THE Z VALUES **
3930 !
3940 REM
3950 PEN 1 @ GCLEAR
3960 SCALE -3,7,-2,8
3970 XAXIS 0,.75
3980 YAXIS 0,1
3990 !
4000 LDIR 0
4010 FOR X=1 TO 10 STEP 2
4020 MOVE X*3/4-.2,-.7
4030 LABEL VAL$(X*10)
4040 NEXT X
4050 !
4060 LDIR 90
4070 FOR Y=2 TO 10 STEP 2
4080 MOVE -.25,Y-.1
4090 LABEL VAL$(Y)
4100 NEXT Y
4110 !
4120 MOVE Y9(2)*3/4,Z9(2)
4130 FOR K=2 TO I9
4140 DRAW Y9(K)*3/4,Z9(K)
4150 NEXT K
4160 COPY
```

```
4170 PRINT
4180 PRINT
4190 RETURN
4200 !
4210 !
4220 !
4230 !
4240 ! ** THIS SUBROUTINE IS TO
     DRAW THE RELATIONSHIP BETWE
     EN THE MAX. DEPTH ON THE X-
     Y PLANE
4250 ! AND Z VALUES, BETWEEN THE
      MAX. WIDTH ON THE X-Y PLAN
     E AND Z VALUES **
4260 !
4270 REM
4280 PEN 1 @ GCLEAR
4290 SCALE -3,7,-2,8
4300 XAXIS 0,.75
4310 YAXIS 0,1
4320 !
4330 LDIR 0
4340 FOR X=1 TO 10 STEP 2
4350 MOVE X*3/4-.1,-.7
4360 LABEL VAL$(X*2)
4370 NEXT X
4380 !
4390 LDIR 90
4400 FOR Y=2 TO 10 STEP 2
4410 MOVE -.25,Y-.1
4420 LABEL VAL$(Y)
4430 NEXT Y
4440 !
4450 MOVE Y9(1)*3/4,Z9(1)
4460 FOR K=1 TO I9
4470 DRAW Y9(K)*3/4,Z9(K)
4480 NEXT K
4490 COPY
4500 PRINT
4510 PRINT
4520 RETURN
4530 !
4540 !
4550 !
4560 ! ** THIS SUBROUTINE IS TO
     DRAW THE RELATIONSHIP BETWE
     EN THE COMMON WORK SPACE VO
     LUME
4570 ! AND THE DISTANCE OF TWO R
     OBOTS **
4580 !
4590 REM
4600 PEN 1 @ GCLEAR
4610 SCALE -3,7,-2,8
4620 XAXIS 0,.75
4630 YAXIS 0,1
4640 !
4650 LDIR 0
4660 FOR X=1 TO 8 STEP 2
4670 MOVE X*3/4-.1,-.7
```

```
4680 LABEL VAL$(X+S(1))
4690 NEXT X
4700 !
4710 LDIR 90
4720 FOR Y=2 TO 10 STEP 2
4730 MOVE -.25,Y-.2
4740 LABEL VAL$(Y*40)
4750 NEXT Y
4760 !
4770 MOVE Y9(2)*3/4,V9(2)/40
4780 FOR K=2 TO N1
4790 DRAW Y9(K)*3/4,V9(K)/40
4800 NEXT K
4810 COPY
4820 PRINT
4830 PRINT
4840 RETURN
4850 !
4860 !
4870 !
4880 ! ** THIS SUBROUTINE IS TO
     DRAW THE COMMON WORK AREA O
     N THE X-Y PLANE FOR GIVEN Z
      VALUES **
4890 !
4900 REM
4910 PRINT "DISTANCE OF TWO ROBO
     TS=";S(I1)
4920 PRINT "Z-COORDINATE=";Z9(M1
     )
4930 M9=M9+5
4940 PEN 1 @ GCLEAR
4950 SCALE -3,7,-2,8
4960 XAXIS 0,.75
4970 YAXIS 0,1
4980 MOVE 0,0
4990 !
5000 LDIR 0
5010 FOR X=2 TO 8 STEP 2
5020 MOVE X*3/4-.1,-.7
5030 LABEL VAL$(X)
5040 NEXT X
5050 !
5060 LDIR 90
5070 FOR Y=2 TO 10 STEP 2
5080 MOVE -.25,Y-.1
5090 LABEL VAL$(Y)
5100 NEXT Y
5110 !
5120 MOVE X1(1)*3/4,X9(1)
5130 FOR I=1 TO J1
5140 DRAW X1(I)*3/4,X9(I)
5150 NEXT I
5160 MOVE X2(1)*3/4,X9(1)
5170 FOR I=1 TO J2
5180 DRAW X2(I)*3/4,X9(I)
5190 NEXT I
5200 MOVE X3(1)*3/4,X9(1)
5210 FOR I=1 TO J1
5220 DRAW X3(I)*3/4,X9(I)
```

```
5230 NEXT I
5240 MOVE X4(1)*3/4,X9(1)
5250 FOR I=1 TO J2
5260 DRAW X4(I)*3/4,X9(I)
5270 NEXT I
5280 COPY
5290 PRINT
5300 RETURN
5310 END
```